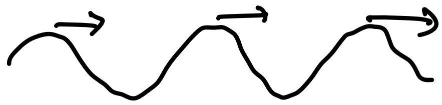
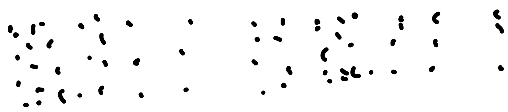
Day 2 - Interference

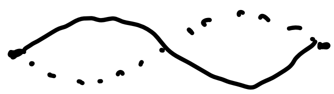
What are some examples of waves?

- ↳ water
- ↳ sound
- ↳ guitar string

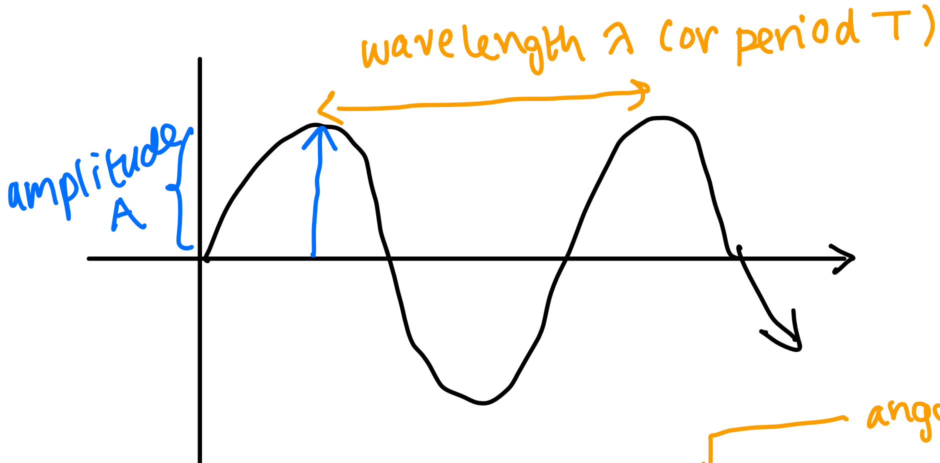
All of these are periodic displacements from equilibrium.

water: 

sound: 

guitar string: 

Sine wave:



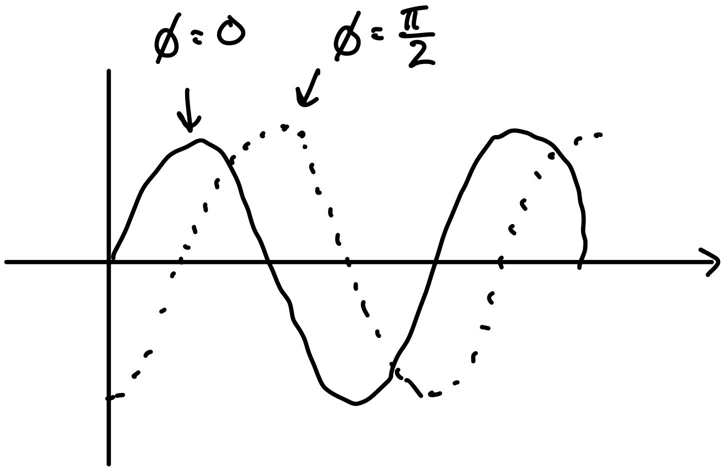
angular frequency

units: rad/s

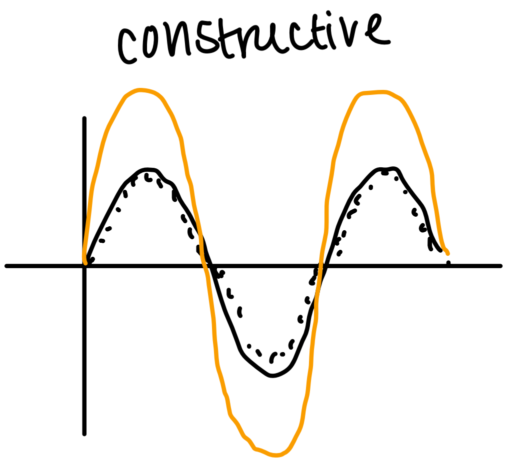
if periodic in time: $\omega = \frac{2\pi}{T}$

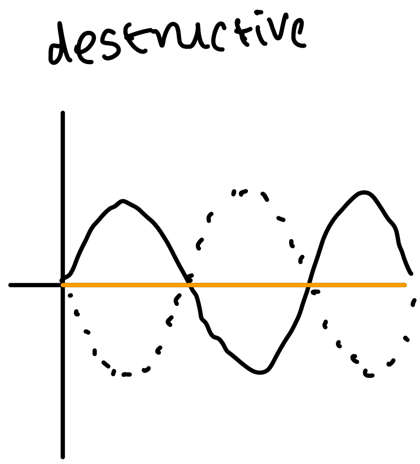
if periodic in space: $\omega = 2\pi \frac{v}{\lambda}$

Equation: $y = A\sin(\omega x + \phi)$ — ϕ: phase



Interference:

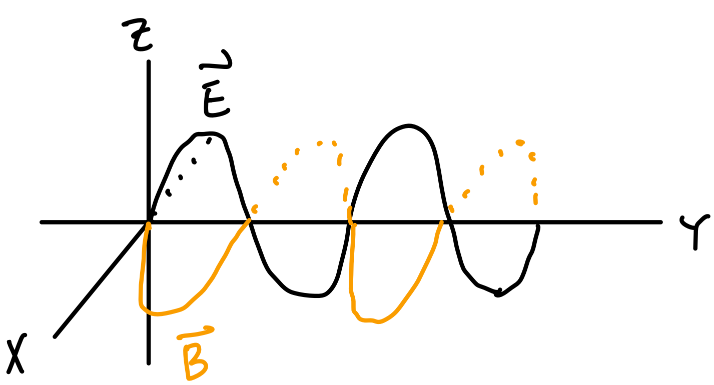




★ Interference is the hallmark of waves ★

↳ if you see interference, you should think "waves"

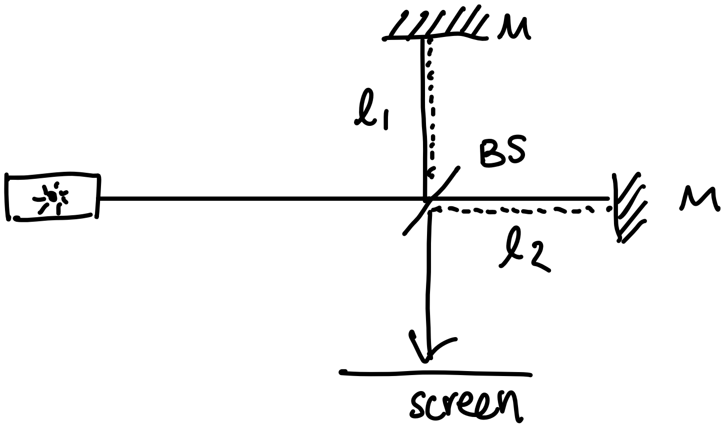
Light is a wave (EM wave)



$\vec{v} = \vec{E} \times \vec{B}$

What do we not expect to behave like waves?
↳ particles  can't have two existing in the same place at once!

Michelson interferometer



conditions for interference?

total constructive: $l_1 = l_2 + \frac{\lambda}{2} n$, $n \in \mathbb{Z}$

total destructive: $l_1 = l_2 + \frac{\lambda}{4}(2n+1)$, $n \in \mathbb{Z}$

e.g. $l_1 = l_2 \pm \frac{\lambda}{4}$

$l_1 = l_2 \pm \frac{3\lambda}{4}$

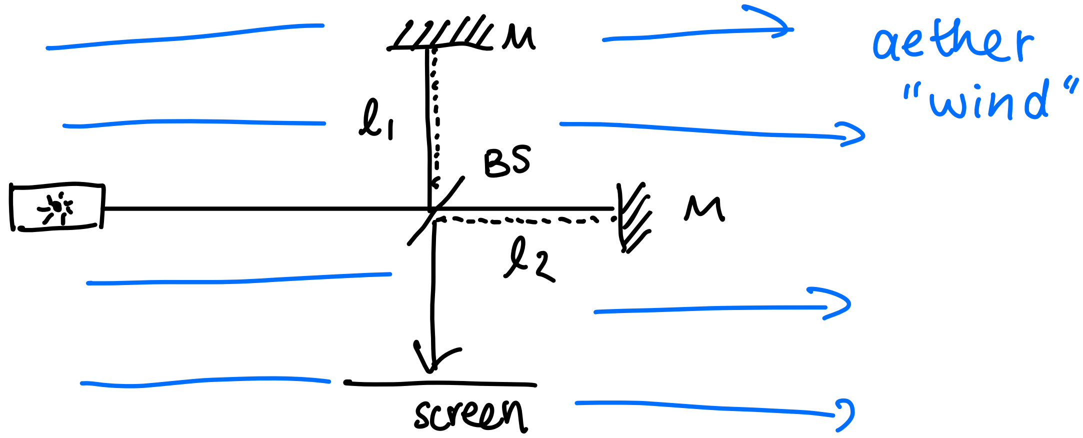
Historic uses of Michelson interferometers:

① Michelson-Morley experiment (1887)

Waves propagate through a medium.
↳ ex: water waves → water
↳ ex: sound waves → air molecules

what medium does light propagate through?

1800's physicists: the luminiferous aether



Hypothesis: the light traveling in the horizontal path (l_2) will take longer because the slow down from traveling against the wind is greater than the speed up of traveling with the wind.

Result: no difference (to within experimental error)

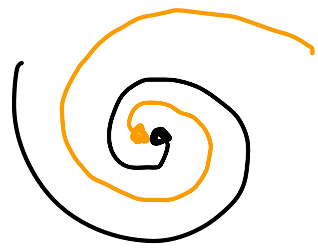
This was the first experimental evidence against the existence of the aether...
AND initiated a line of research which led to the conception of special relativity by Einstein in 1905

② LIGO (Laser Interferometer GW observatory)

Einstein developed the theory of general relativity in 1915, which explains the warping of spacetime by massive objects like black holes.

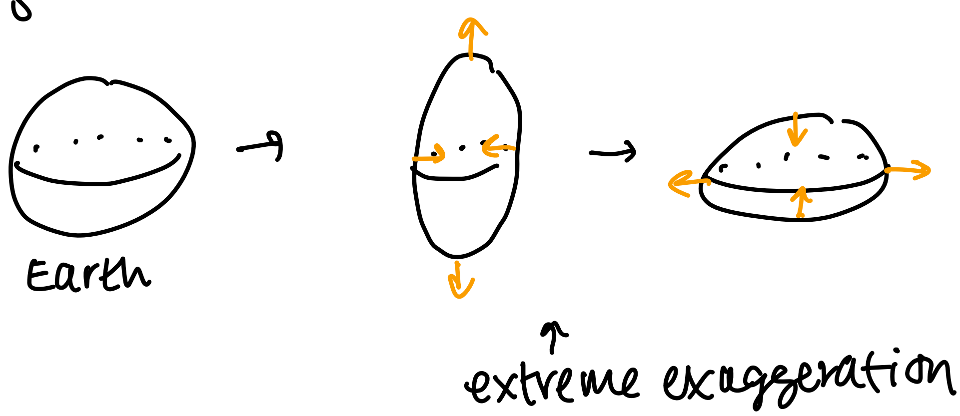
If you have two massive objects circling each other, this will send ripples in spacetime outwards.

Ex. binary black hole merger

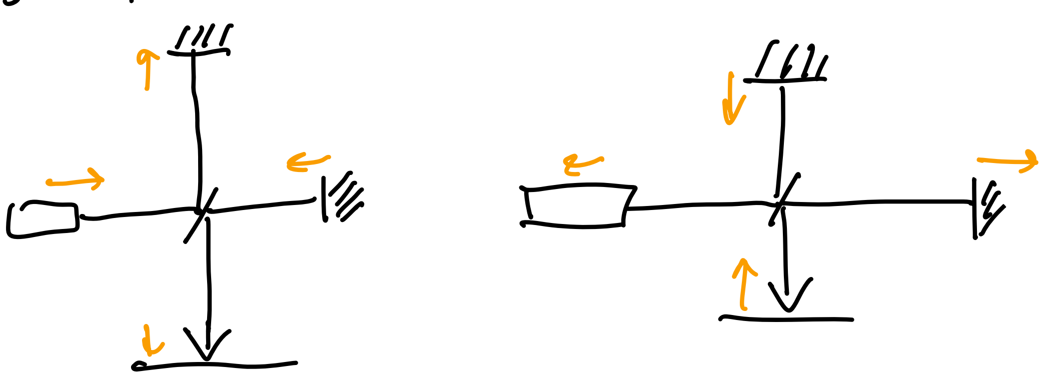


Such an event would send GW outwards, which would eventually pass through Earth.

A GW is a wave — displacement from equilibrium. These displacements are squeezing and stretching along longitudinal and transverse axes.



Michelson interferometer:



The speed of light is the same in all reference frames and along every direction.

We would expect to see oscillations in the interference strength.

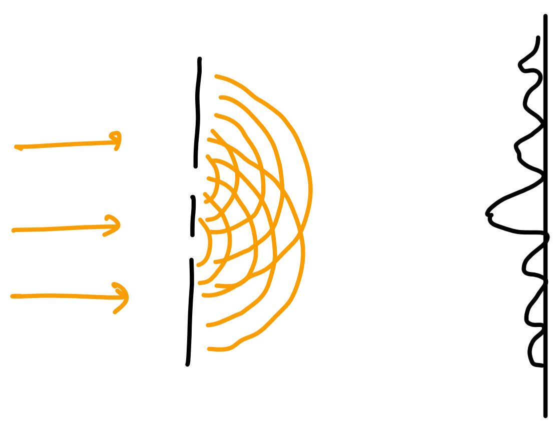
LIGO: Michelson interferometer with 4km arms

↳ first ever detection of GW (binary black hole merger) in 2015 ... 100 years after the birth of general relativity!

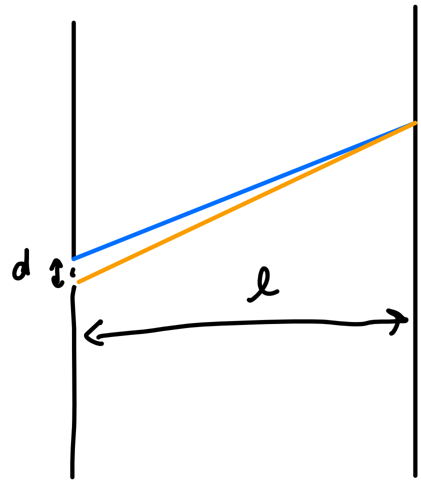
Young's Double Slit Experiment (1801)

(Matt will describe in detail)

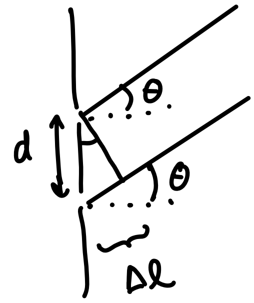
This experiment confirmed that light exhibited wave like behavior.



Interference condition:



in the limit $l >> d$, then we have approximately the same angle from each slit.



$$\Delta l = d\sin\theta$$

constructive: $d\sin\theta = m\lambda \quad m \in \mathbb{Z}$

destructive: $d\sin\theta = \frac{\lambda}{2}(2m+1) \quad m \in \mathbb{Z}$

What I have just told you is true for a beam of light... what about for a single photon?

Interference

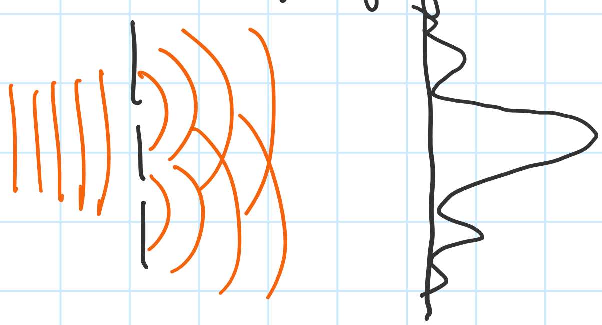
Sunday, January 12, 2025 11:19 AM

- Interference is the hallmark of waves! (Cora)
- Probability waves (Matt)
  - Quantum vs classical coins
- Evolution of waves: gates and the Bloch sphere (Agi)
  - No hidden variable?
  - No-cloning theorem?
- Physical implementation of gates (Matt)

II. Double-slit experiment

a) Why waves? (5 min)

Consider light going through two slits



Demo?

Interference pattern emerges!

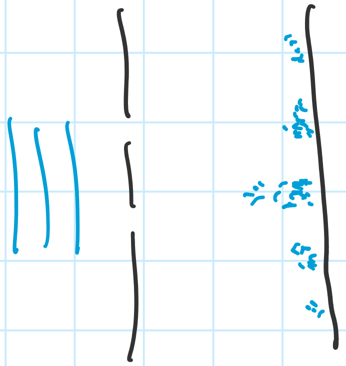
This is b/c the electric fields are adding and subtracting together,

$E_1(x) + E_2(x)$

However, the measurable quantity is not E, but rather the intensity $I \propto |E|^2$

b) Well, we could do a similar thing w/ particles (say, an electron beam) (5 min)

Or single photons



Wave-particle duality:
Particle like in one place
Wave pattern over many shots

In this case, we get a similar looking interference pattern

⇒ Wave phenomena

However, the pattern arises asymptotically, that is, any particular electron could show up anywhere (can't predict), but in the limit of a lot of electrons the distribution emerges.

⇒ probabilistic theory

So in keeping with this train of thought we might say:

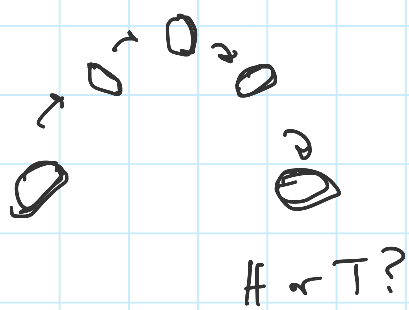
1) Wave theory
2) Measurable quantity is some probability distribution

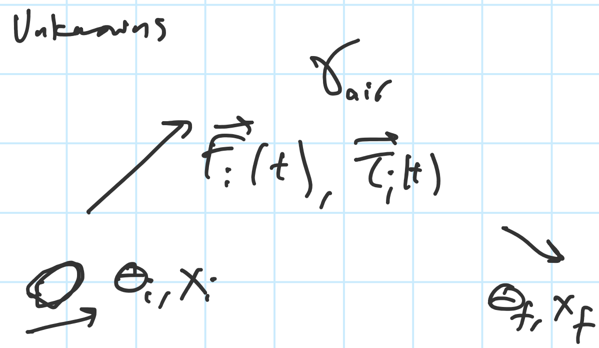
2) Measurable quantity is some probability distribution

3) The measurable quantity should be related to $|\text{wave}|^2$

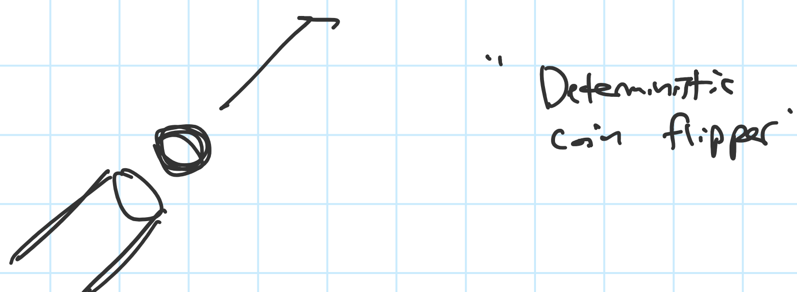
c) Quantum vs classical probability (10 min)

It's important to note that this randomness does not arise because we don't "know enough" about our system. That is classical randomness.





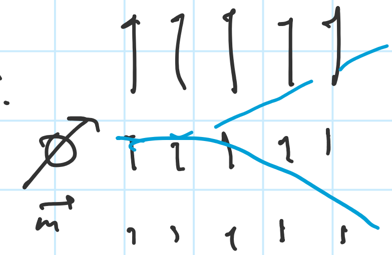
However, in theory if we knew everything about our system, we could completely predict the trajectory.



(In practice this is difficult, but the point remains classical randomness is not fundamental)

Quantum randomness is <u>fundamental</u> — you can know everything about the system, its trajectory, its final state. But upon measuring, you cannot predict which result you will get.

Stern-Gerlach:

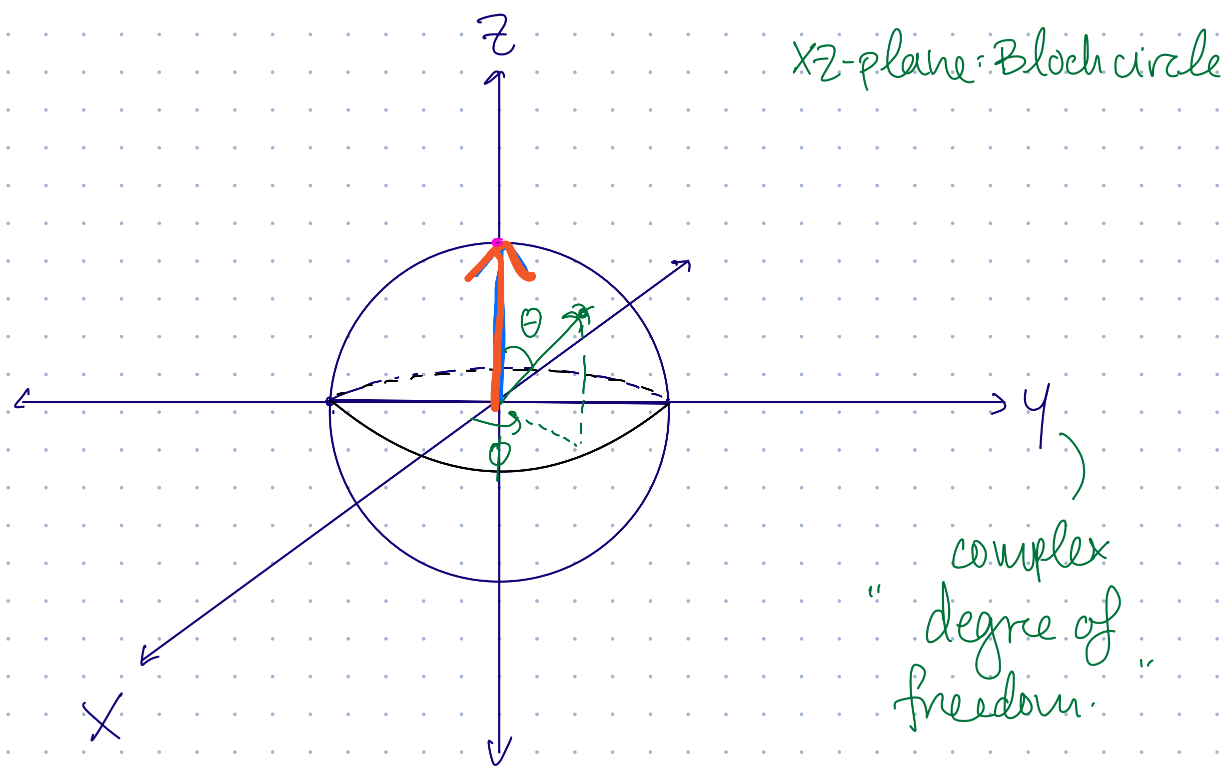


$$|\psi_f\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

On each run, you might get $\uparrow$ or $\downarrow$. But you can't predict which!

$\longrightarrow$ QRNG.

Bloch sphere:



How is an arbitrary qubit on the Bloch sphere defined?

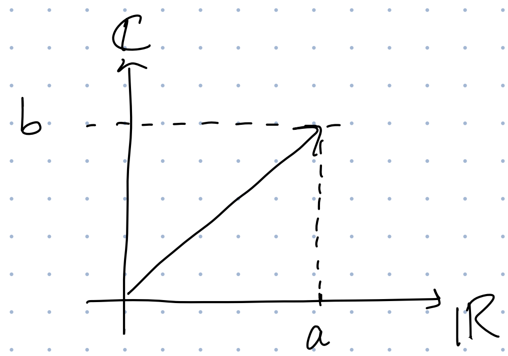
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad \alpha, \beta \in \mathbb{C}$$

Complex numbers: deal w/ imaginary component, "i".

$$z = a + bi$$

$$z^* = a - bi$$

$$|z| = \sqrt{zz^*} = \sqrt{z^* z} = \sqrt{a^2 + b^2}$$



Want, for an arbitrary quantum state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle : \quad |\alpha|^2 + |\beta|^2 = 1.$$

Understanding the Bloch sphere:

A point specified by $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

can be derived in the Cartesian picture:

$$|\psi\rangle = |\alpha| e^{i\theta_\alpha}|0\rangle + |\beta| e^{i\theta_\beta}|1\rangle$$

$$= e^{i\theta_\alpha}\left(|\alpha||0\rangle + |\beta| e^{i\theta_\beta - i\theta_\alpha}|1\rangle\right)$$

let $\theta_\alpha = \gamma$ and $\theta_\beta - \theta_\alpha = \phi$

$$= e^{i\gamma}\left(|\alpha||0\rangle + |\beta| e^{i\phi}|1\rangle\right)$$

We know that $|\alpha|^2 + |\beta|^2 = 1$

$\therefore \; |\alpha| = \cos(\theta/2)$ and $|\beta| = \sin(\theta/2)$

so

$$= \underbrace{e^{i\gamma}}_{\text{global phase}}\left(\cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\phi}|1\rangle\right)$$

$$= \left(\cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle\right).$$

where $0 \le \theta \le \pi$ and $0 \le \phi \le 2\pi$

corresponding spherical coordinates are: $\begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$

Recall: X, Z, I, H.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{(Pauli-X)}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{(Pauli-Z)}$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

X-gate: "bit flip"

Z-gate: "phase flip"

$$Z|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$$

↑ PHASE

Global vs. relative phase:

global := not observable (i.e. ∄ no measurement that reveals the phase).

e.g. $-|1\rangle$

relative := observable

e.g. $|-\rangle, |+\rangle$.

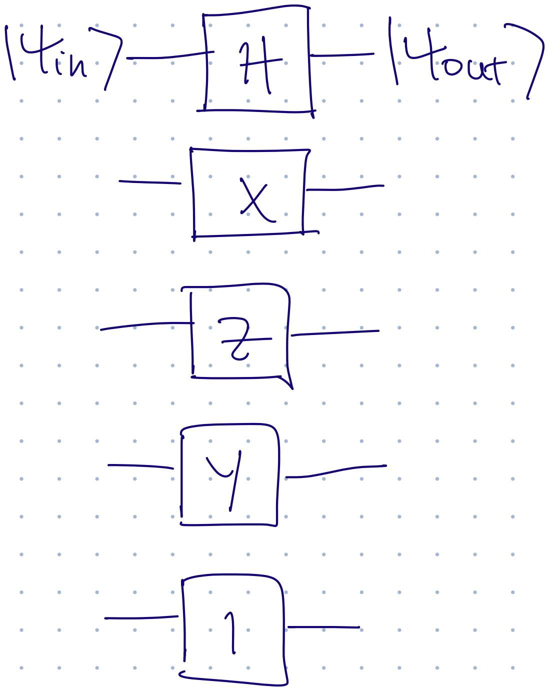
Y-gate: $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$$Y|0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = i \begin{pmatrix} 0 \\ 1 \end{pmatrix} = i|1\rangle$$

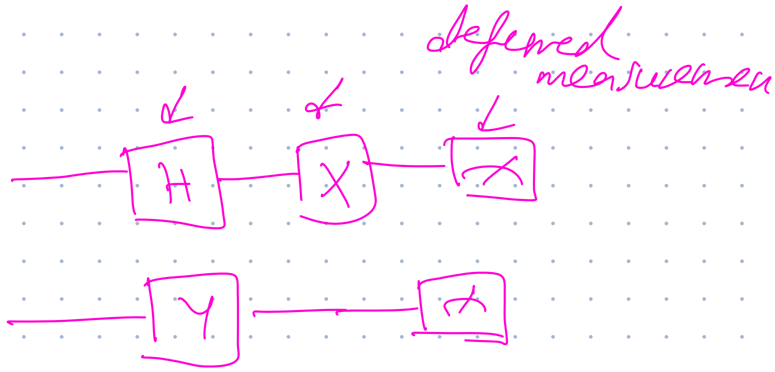
$$Y|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = -i \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -i|0\rangle$$

"bitflip" <u>and</u> a "phaseflip"!

Quantum Circuit:

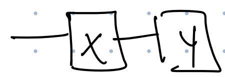


single-qubit quantum gates.



*emphasize that partial ordering of gates is reversed when writing as an equation.

$XY|\psi\rangle \rightarrow$



Simple example of interference:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$H|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$$

in the computational basis:

$$= \alpha\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) + \beta\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$$

$$= \frac{\alpha+\beta}{\sqrt{2}}|0\rangle + \frac{\alpha-\beta}{\sqrt{2}}|1\rangle$$

interference: each amplitude is interfering w/ the other!

Two-qubit quantum states:

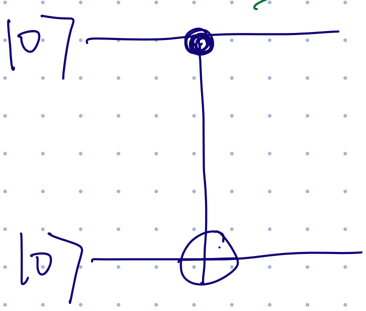
$$|0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

↑ tensor product.

tensor product: a vector operation that increases dimension

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \otimes \begin{pmatrix} \gamma \\ \eta \end{pmatrix} = \begin{pmatrix} \alpha\gamma \\ \alpha\eta \\ \beta\gamma \\ \beta\eta \end{pmatrix}$$

CNOT: "controlled-NOT" gate:



→ "control" bit
if equal to 1: activate bottom qubit
else: do nothing.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Universal set of gates: can simulate <u>any</u> computation.

e.g. $\{H, CNOT, S, T\}$

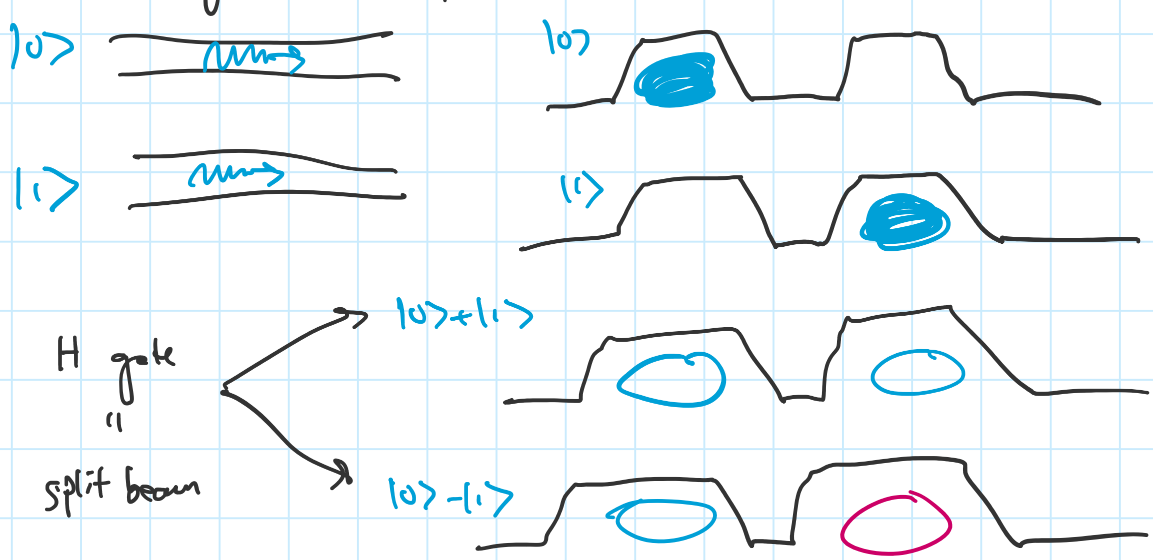
Phase gate $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$

$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$

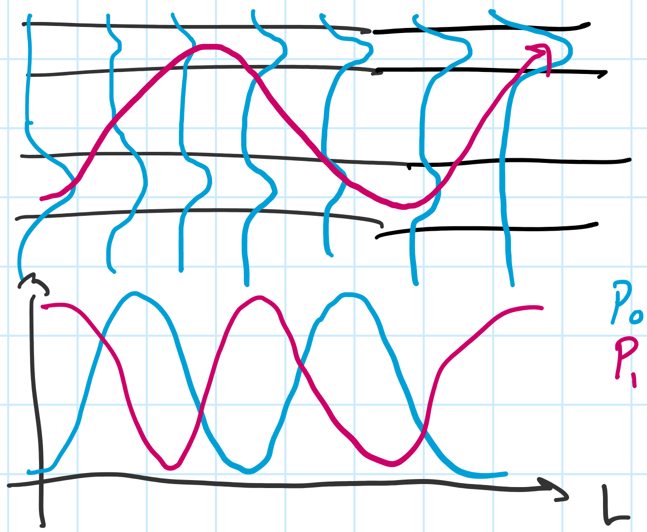
IV Physical implementation of gates

a) Light

"Dual-rail qubit" (path encoding)



How do you split a beam? Use a ... beam splitter

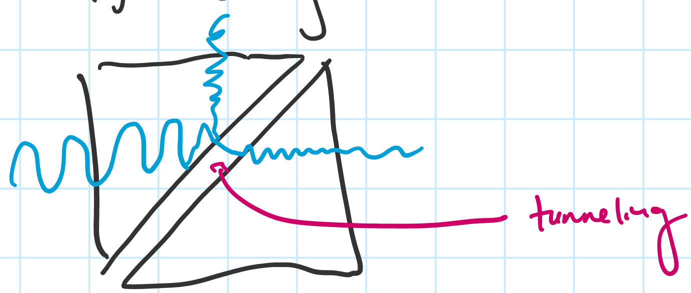


"Rabi oscillations"

"Coupled" when placed close together

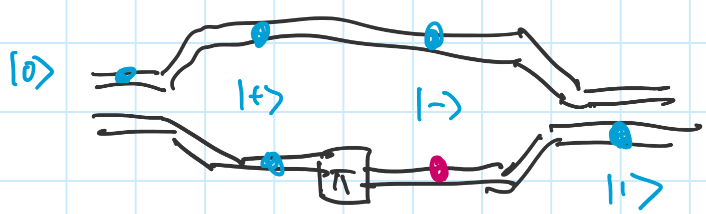
→ light slowly transfers

Demo?



BONUS

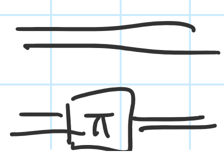
- Mach-Zehnder interferometer



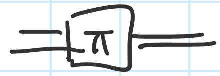
$|0\rangle \to |1\rangle$

$|1\rangle \to |0\rangle$

X gate!

But what is  anyway?

But what is



anyway.

$$|0\rangle \longrightarrow |0\rangle$$
$$|1\rangle \longrightarrow e^{i\pi}|1\rangle = -|1\rangle$$

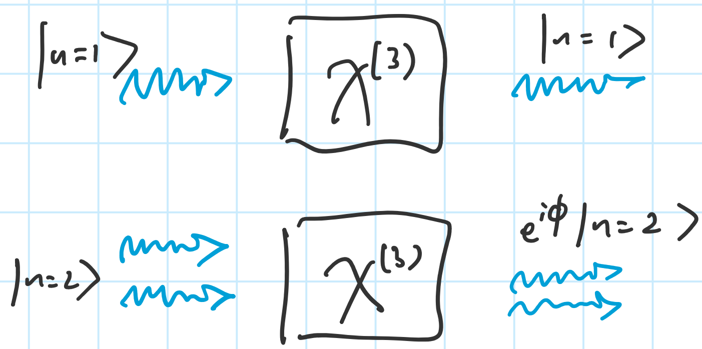
Z gate!

Physically, implements identity

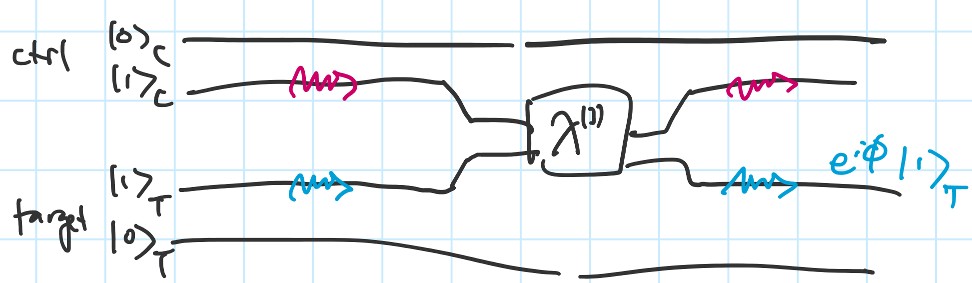
$$X = HZH$$!

- CNOT BONUS

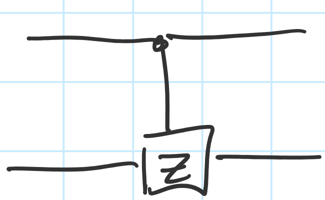
"Kerr-effect" = intensity-dependent index



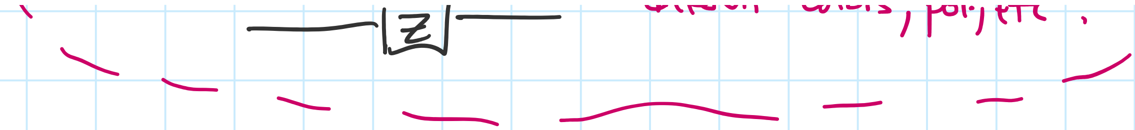
very hard in practice!



||



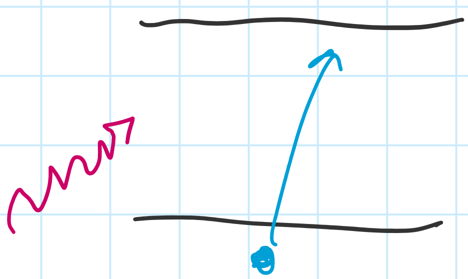
To separate out ctrl + target, maybe different colors, pol, etc.

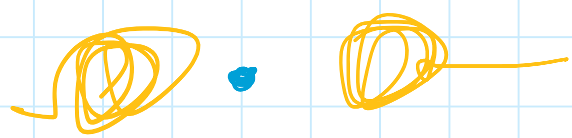


Main point:

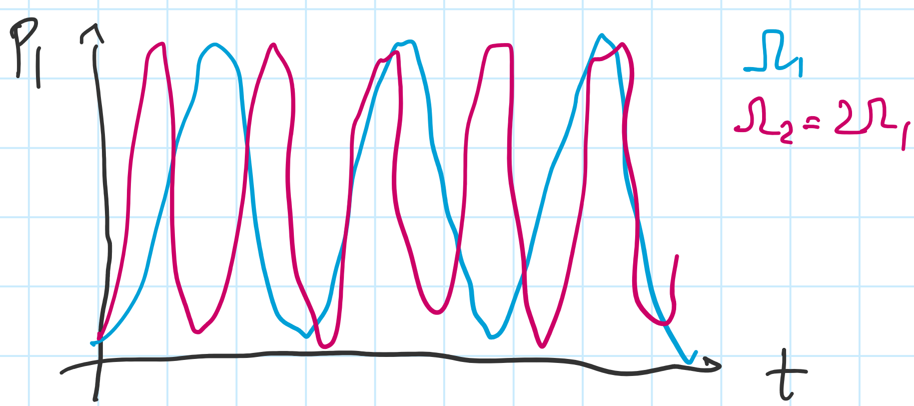
- Single-qubit gates realized by "coupling" two states
- Qubit state oscillates, which gate determined by duration of coupling
- Two-qubit gates realized by perturbing the state of the target qubit through strong interaction between two qubits

b) Atoms (and other two-level systems)





$$f_{drive} = \frac{\Delta E}{h}$$ match energy!



"Rabi frequency" proportional to strength of interaction

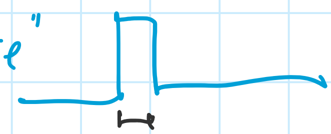
↳ $\Omega \propto \vec{d} \cdot \vec{E}$

qubit more sensitive to E/B → stronger interaction

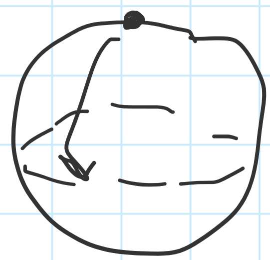
driving field bigger → stronger interaction

$$P_{0\to1} = \sin^2\left(\frac{\Omega t}{2}\right)$$

"$\frac{\pi}{2}$ - pulse"

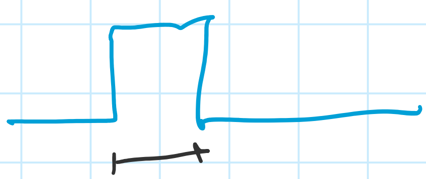


$t = \pi/2\Omega$

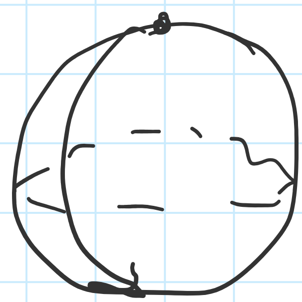


~H

"π pulse"



$t = \pi/\Omega$



X

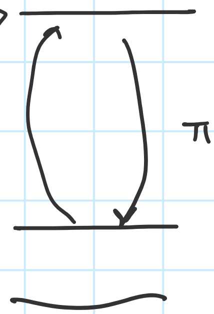
BONUS

• CNOT

"Rydberg blockade"

|r⟩

π π

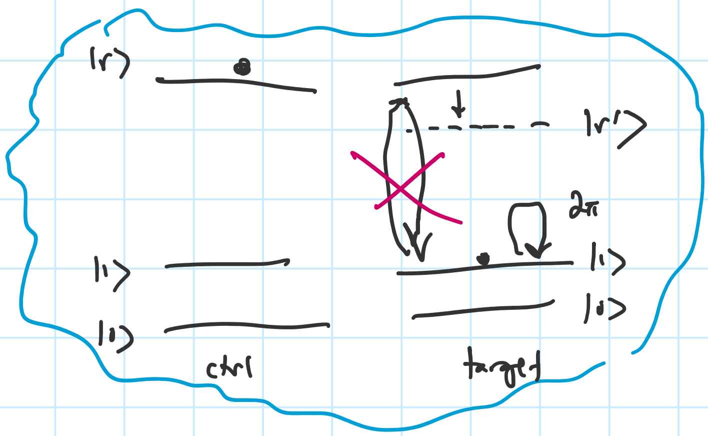


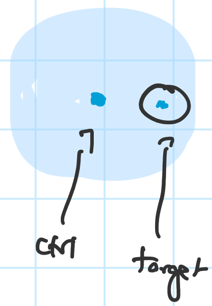
|1⟩

|0⟩

"2π pulse"

$|1\rangle \to |r\rangle \to -|1\rangle$





$|1\rangle_T \rightarrow |1\rangle_T$