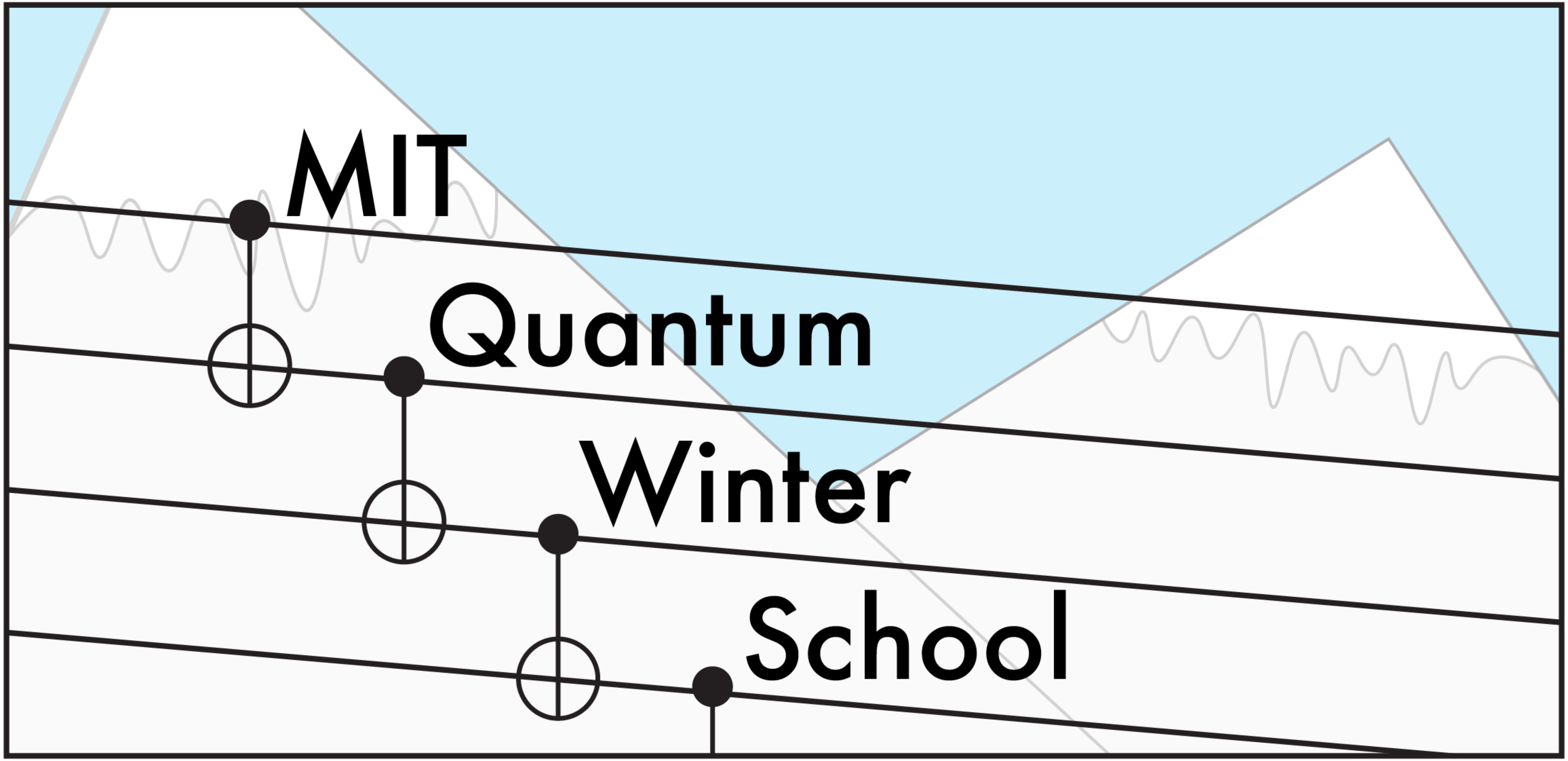

Lecture 1: Superposition

This course: a Quantum of Quantum Computing

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Quantum *mechanics* is about understanding a world that is hard to see.

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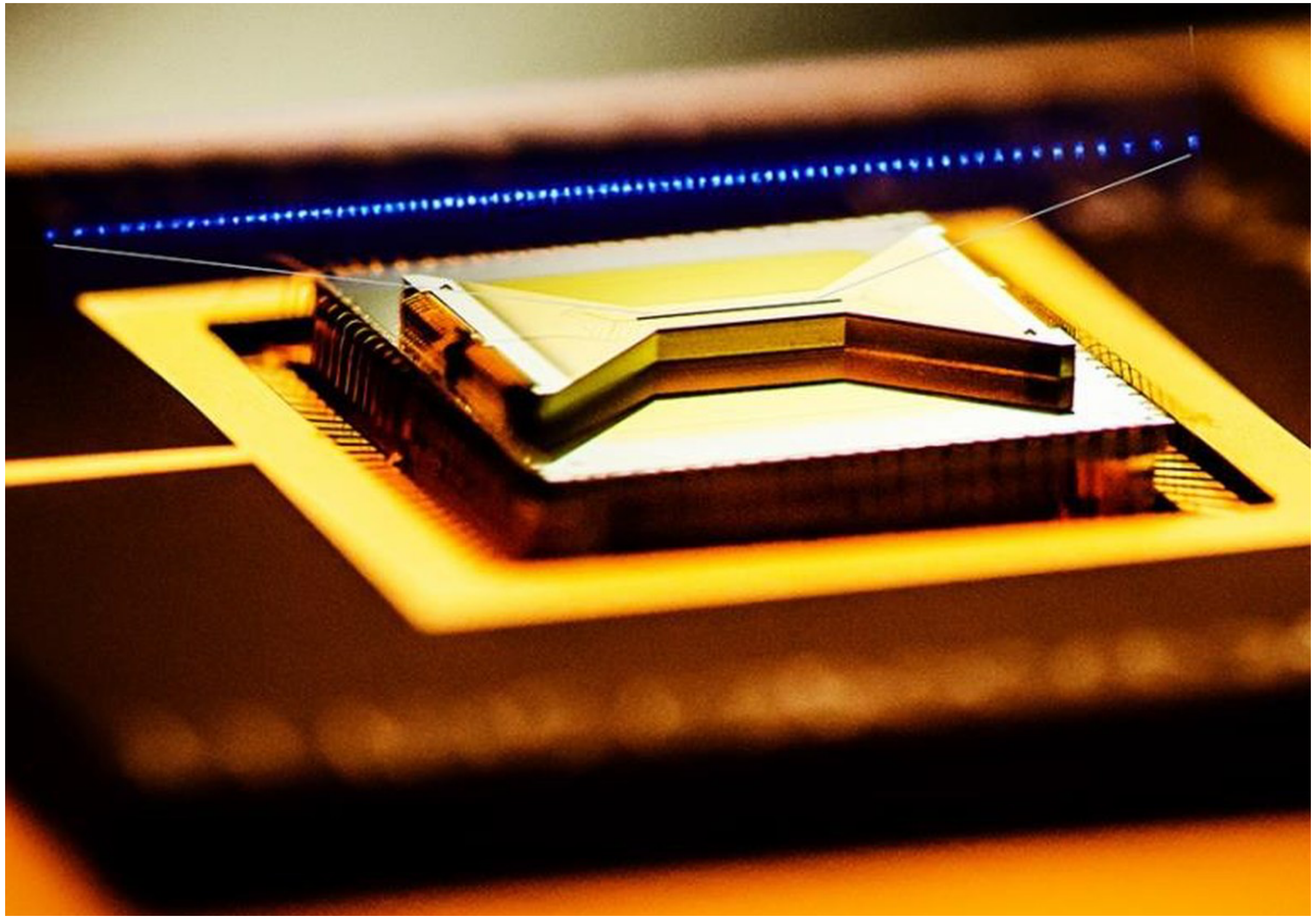

The linear ion-trap on an IonQ chip. https://ionq.com/technology

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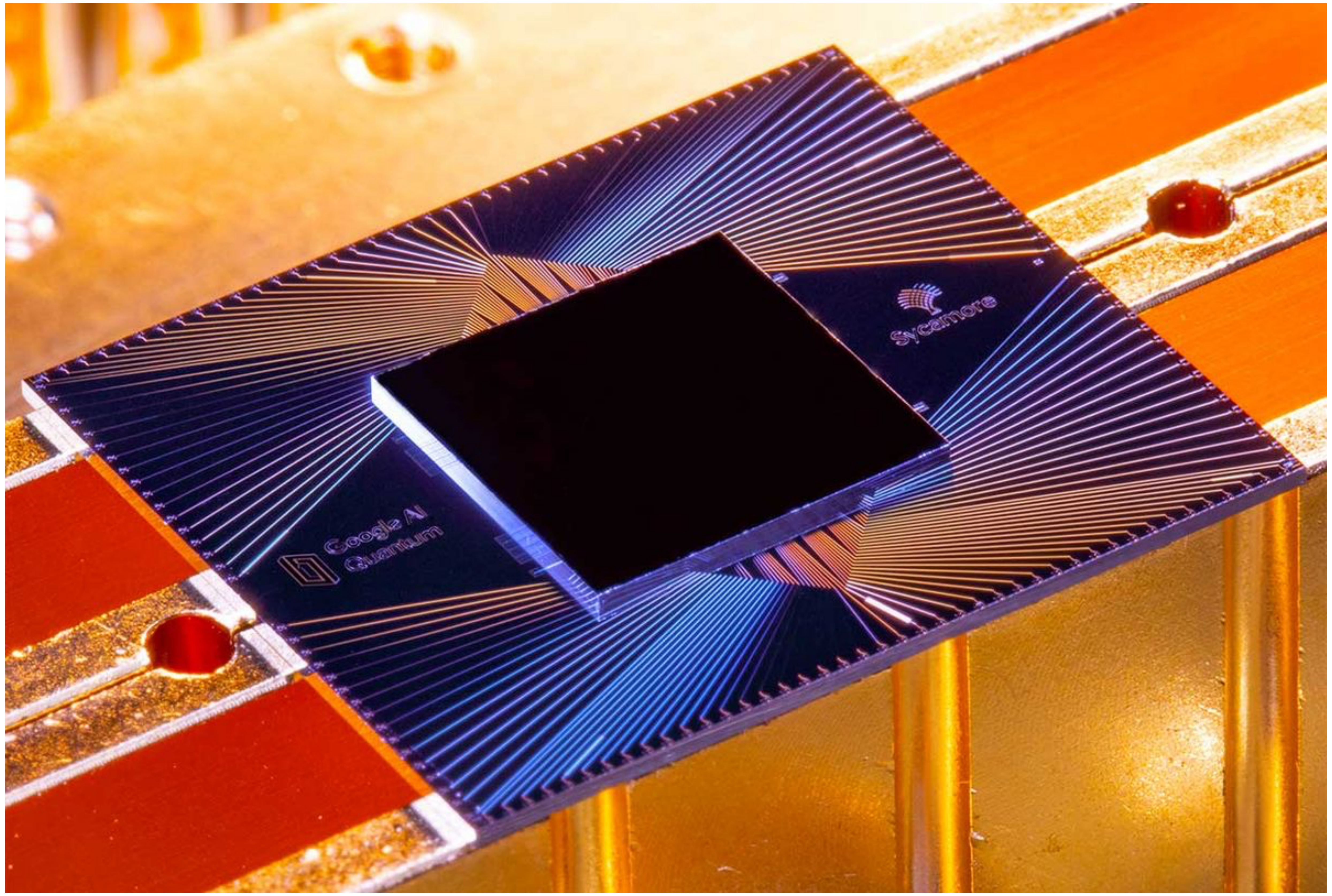

The Google Sycamore superconducting processor. https://spectrum.ieee.org/googles-quantum-computer-exponentially-suppress-errors

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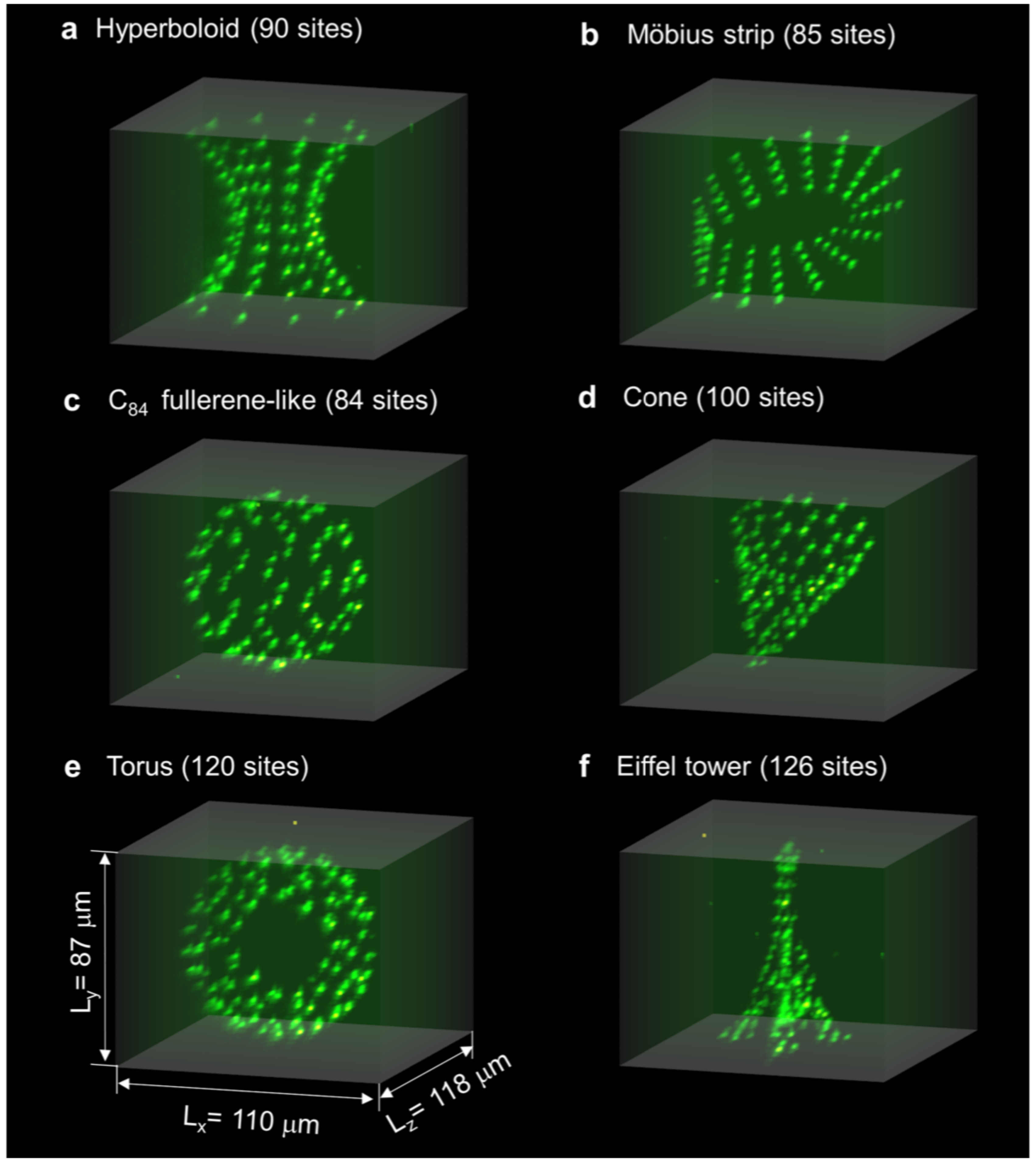
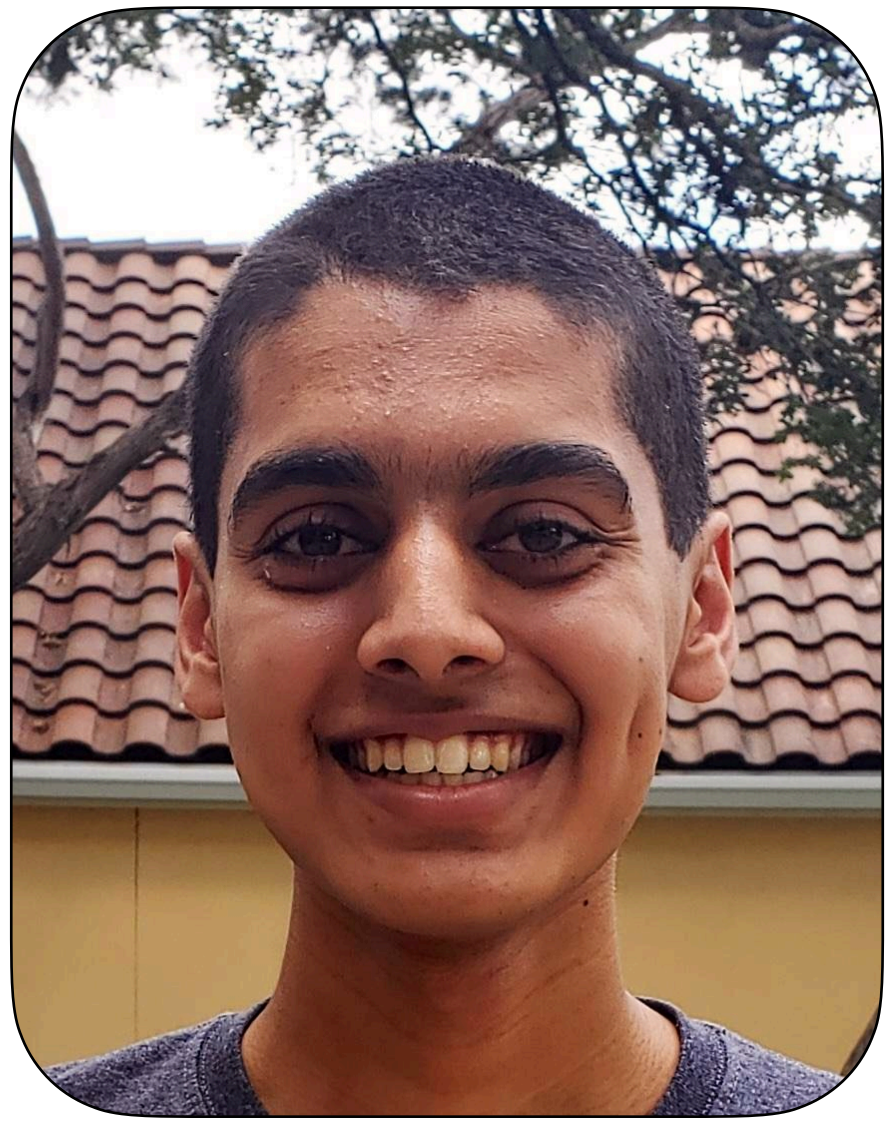


https://arxiv.org/abs/1712.02727: "Single atom fluorescence in 3d arrays. (a-f) Maximum intensity projection reconstruction of the average fluorescence of single atoms stochastically loaded into exemplary arrays of traps. The x,y,z scan range of the fluorescence is indicated and is the same for all the 3d reconstructions."

Instructors

Instructors

Cora Barrett (she/her)
PhD student, MIT Physics



Om Joshi (he/him)
PhD student, MIT RLE

Matthew Yeh (he/him)
PhD student, Harvard SEAS

Ági Villányi (they/them)
PhD student, MIT CSAIL

Schedule

Schedule

Tues: Superposition	Wed: Interference	Thurs: Entanglement	Friday: Applications
Lecture 10am - 12pm 4-149	Lecture 10am - 12pm 4-149	Lecture 10am - 12pm 4-149	Lecture 10am - 12pm 4-149
Lunch 12pm - 1pm	Lunch 12pm - 1pm	Lunch 12pm - 1pm	Lunch 12pm - 1pm
Problem Solving Session 1pm - 3pm 4-149	Problem Solving Session 1pm - 3pm 4-149	Problem Solving Session 1pm - 3pm 4-149	Problem Solving Session and Keynote Talks 1pm - 3pm 4-149

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1900: Planck and Blackbody Radiation

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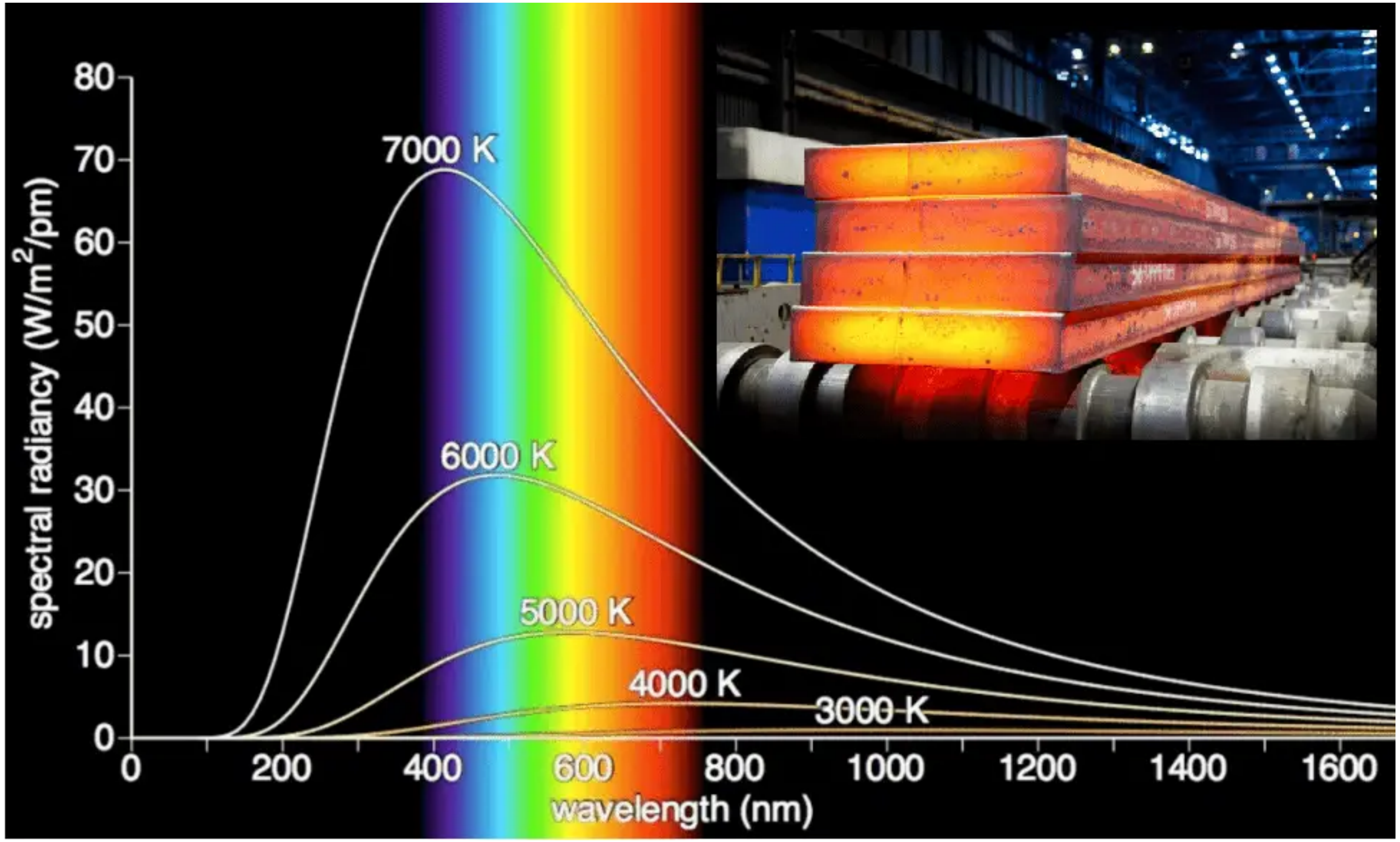

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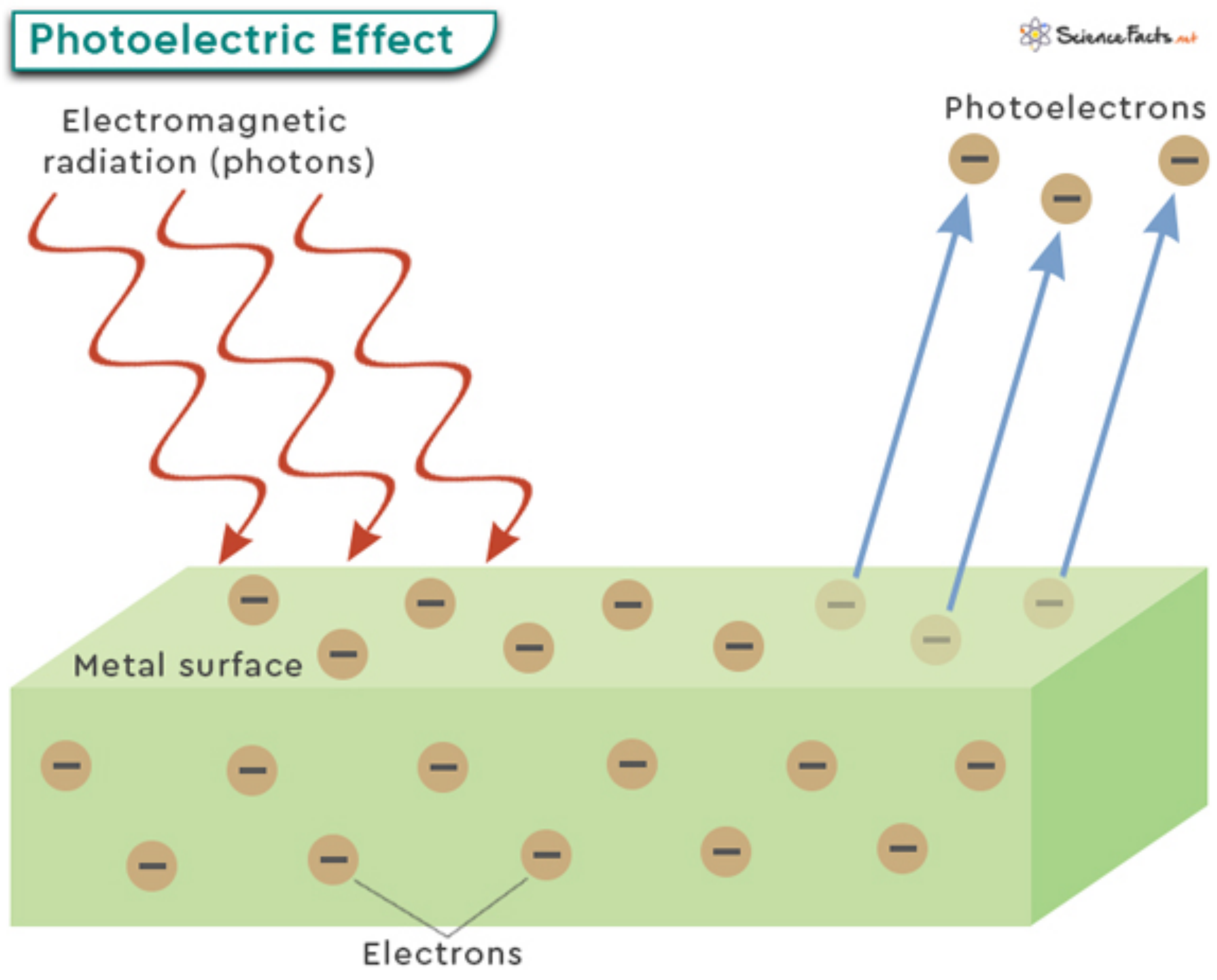

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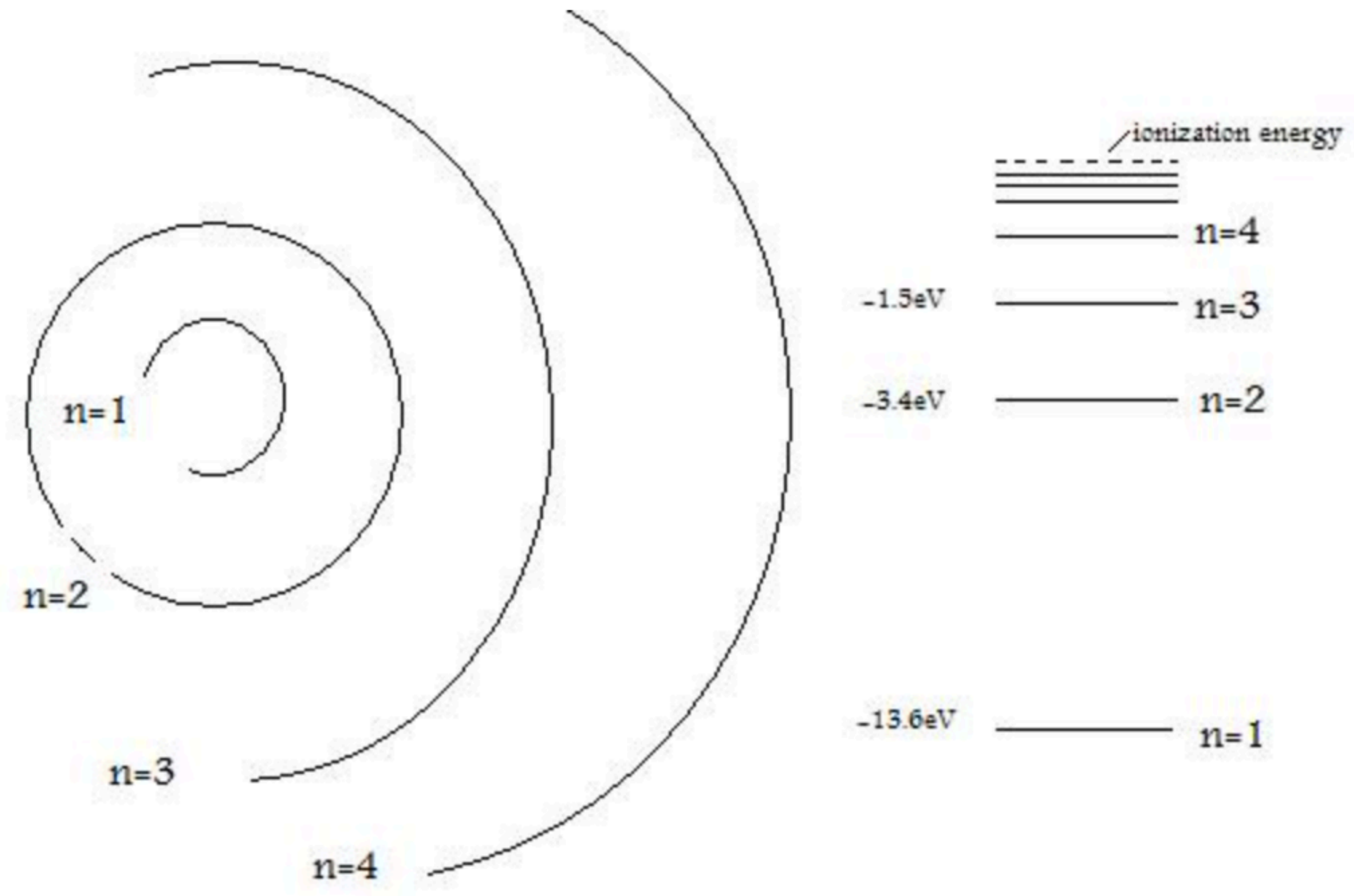

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Stern-Gerlach Experiment

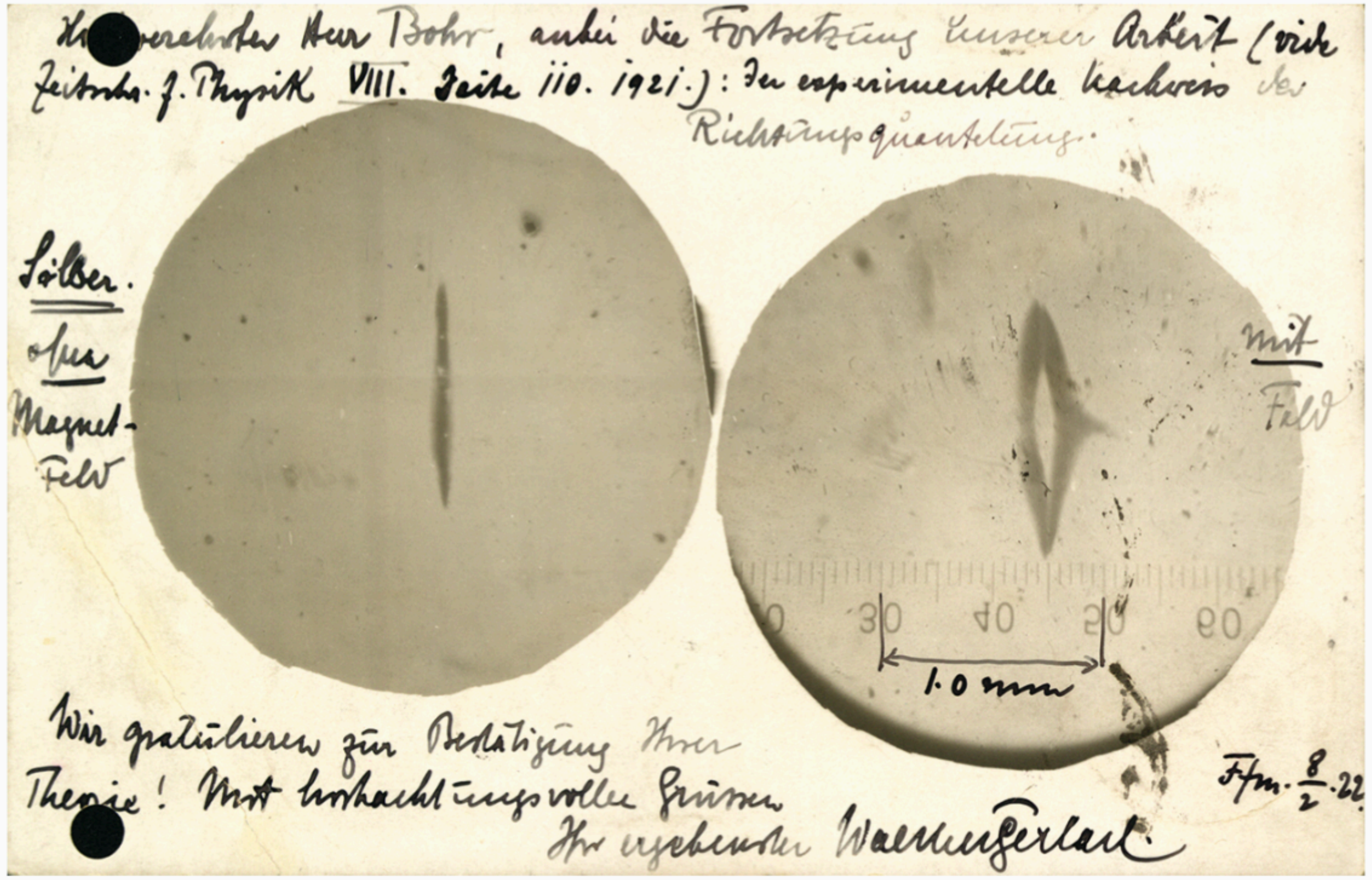

Fig. 1. Walther Gerlach sent this postcard to Niels Bohr, which says in German: "Attached is the experimental proof of spatial quantization (silver without and with field). We congratulate you on the confirmation of your theory.".

How did we get here?

1900: Planck and Blackbody Radiation

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- **1964:** Bell’s Theorem

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- **1964:** Bell’s Theorem
- **1972:** CHSH Experimental Violation
- **1981:** Feynman and Quantum Simulation
- **1994:** Peter Shor and Factoring

Why are we doing this?

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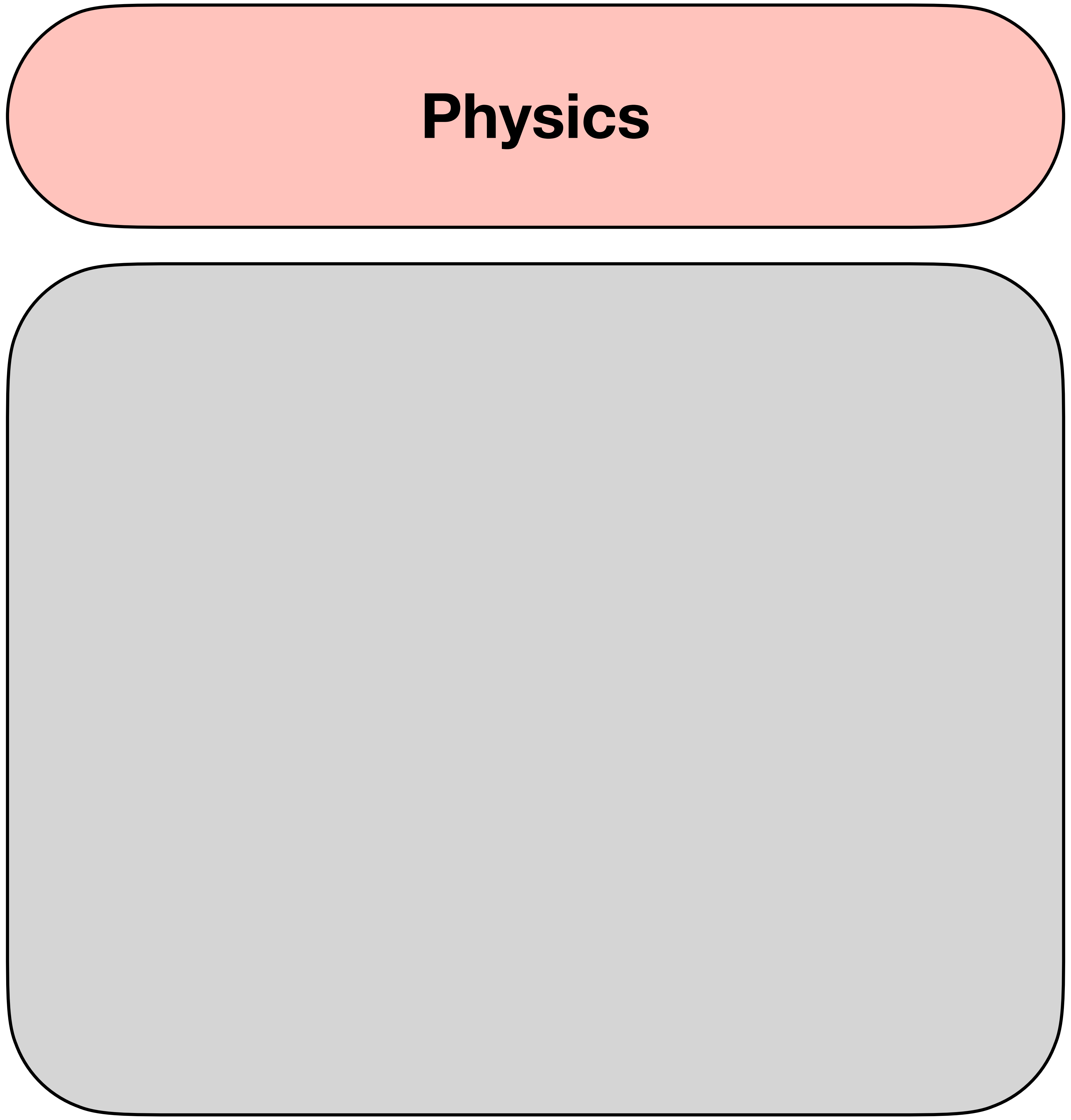
Physics

Why are we doing this?



Why are we doing this?

Physics

- **Quantum simulation**: approximate quantum dynamics on a computational device.
  - Quantum chemistry
  - Engineering new materials
  - Fundamental many body physics discoveries
- **Quantum sensing**: using quantum bits (qubits) for precision measurement.

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Computer Science

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Computer Science

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Computer Science

- **Computability**: The Extended-Church Turing Thesis claims that every reasonable computer that can be built physically can be simulated by a Turing machine. Is this true? Cryptography: more secure communication protocols (quantum cryptography), new challenges of developing quantum-safe protocols (post-quantum cryptography).
- **Algorithms**: new models of computation and new tools for both quantum and classical algorithms.

DiVincenzo’s Criteria

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1. The ability to construct a qubit, physically.

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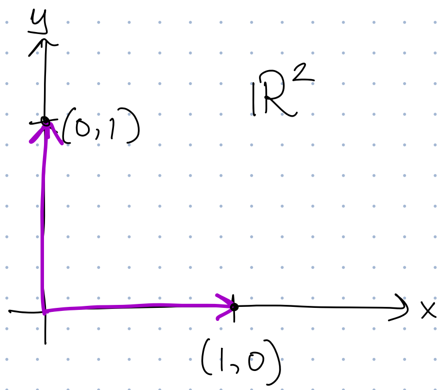
At any given point in time, a ***classical computation*** with an n-bit memory can work with n bits of data, while a quantum computer with an n-qubit memory can work with 2^n bits of data.

The Qubit

- When using quantum mechanics to represent state, we no longer work w/ scalar quantities. Instead, we work with vectors. So, for example, a bit $b \in \{0, 1\}$ can be represented by

$$\vec{b} \in \{\vec{0}, \vec{1}\}$$



A vector is simply when we draw an arrow to a point

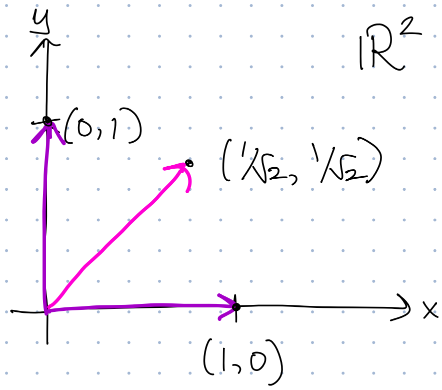
It has both a magnitude (size) and a direction.

To represent vectors, we use a special notation called Dirac notation, which is a short-hand:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

In classical computation, the only possible states are those defined above. In quantum computation, the possible states include the "in-between" ones:



This state is the "equal superposition" state, denoted by $|+\rangle$:

$$|+\rangle = 1/\sqrt{2}\,|0\rangle + 1/\sqrt{2}\,|1\rangle$$

☆Note that this is equivalent to writing:

amplitude

$$|+\rangle = \underbrace{1/\sqrt{2}}_{\text{amplitude}}\begin{pmatrix}1\\0\end{pmatrix} + 1/\sqrt{2}\begin{pmatrix}0\\1\end{pmatrix}$$

$$= \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$$

(This relies on the scalar mult. and add. properties of vectors).

mention that this is a col. vec.

It is important that quantum states are normalized. Namely, their magnitude is always 1.

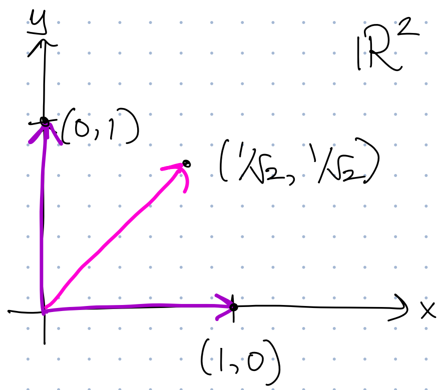
magnitude: of a vector $\vec{v} = \begin{pmatrix}a\\b\end{pmatrix} = \sqrt{a^2+b^2}$

magnitude of $|+\rangle$: $= \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}}$

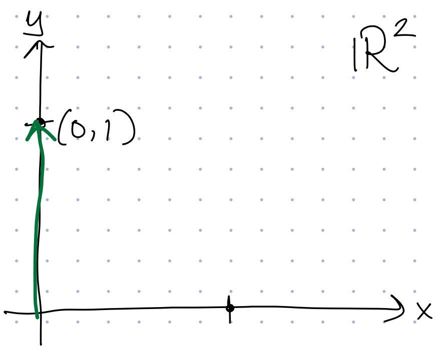
$= 1.$

That is, the sum of the squares of amplitudes should add up to 1.

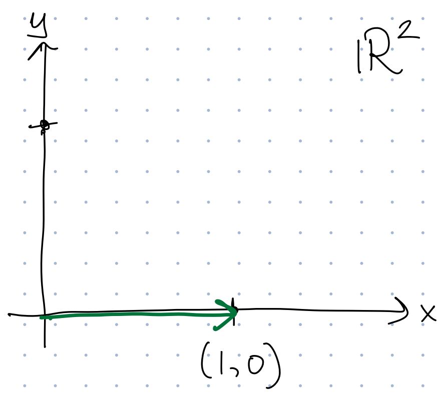
Why is this important? It is important because the amplitudes actually correspond to probabilities.



measurement



OR

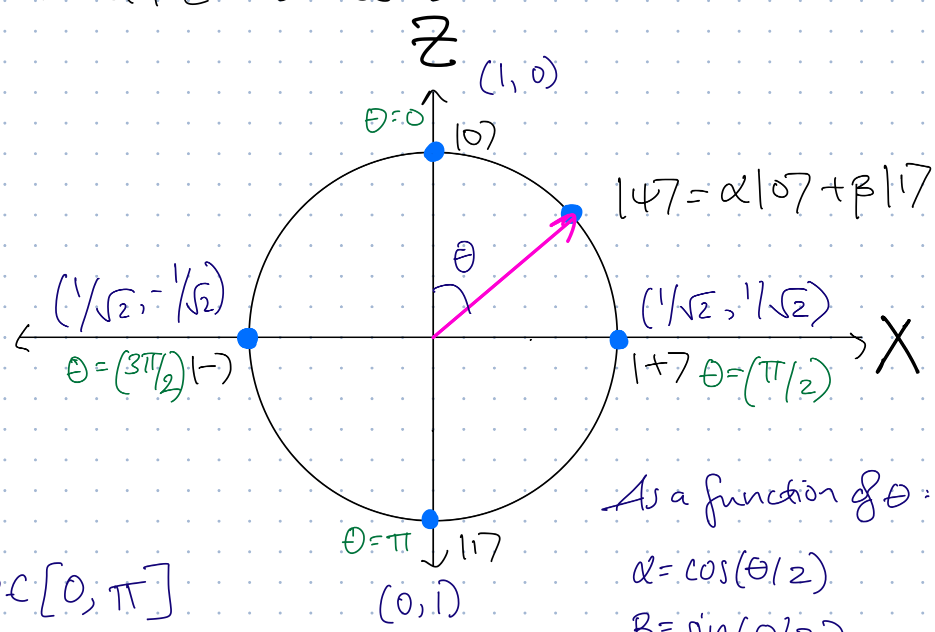


Each of these outcomes happens with probability $(1/\sqrt{2})^2 = 1/2$
$= 50\%$

☆ This is known as the Born Rule.

Probabilities assign fractional values to possible outcomes. All possibilities must add up to 1.

- Another way to visualize the above vectors is using the Bloch coordinates, which uses a coordinate system that is different from the Cartesian coordinates:



$\theta \in [0, \pi]$

As a function of θ:

$\alpha = \cos(\theta/2)$

$\beta = \sin(\theta/2)$.

General qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

"ψ" is from quantum mechanics

$\alpha, \beta \in \mathbb{C}$ (but more on that in lecture 2).

- Now that we have a handle on a quantum state, how can we move from one state to the next?

Using matrices!

A matrix is a "container" for scalar values:

e.g. $\begin{pmatrix} 1 & 2 \\ 7 & 0 \end{pmatrix}$

Matrices can be multiplied onto vectors to form new vectors:

$$\begin{pmatrix} 1 & 2 \\ 7 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$$

4 important matrices for now:

show example matrix mult for each gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : \quad |0\rangle \to |1\rangle, \quad |1\rangle \to |0\rangle$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} : \quad |0\rangle \to |0\rangle, \quad |1\rangle \to -|1\rangle$$

↳ this is a "phase", do not worry about this for now.

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} : \quad |0\rangle \rightarrow |+\rangle, \quad |1\rangle \rightarrow |-\rangle$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} : \quad |0\rangle \rightarrow |0\rangle, \quad |1\rangle \rightarrow |1\rangle$$

Superposition

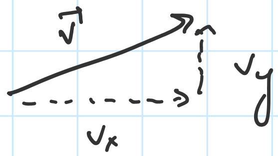
Thursday, January 9, 2025 12:17 AM

- A brief history of quantum mechanics
- Why develop quantum information science?
- Introducing: the qubit
  - Mathematical formalism

Physical implementations of qubits (~5 min)

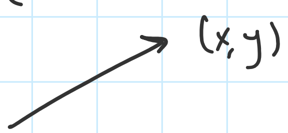
A qubit is a ...

vector? A magnitude and direction?



Hard to imagine encoding information in a moving object...

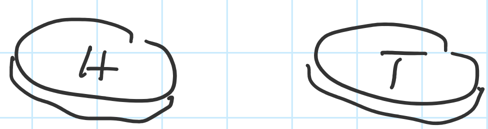
vector? (math)



$\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$

Doesn't give us any physical intuition, however...

vector (math + physics)



$|V\rangle = \alpha|H\rangle + \beta|T\rangle \doteq \vec{v} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

"represented as"

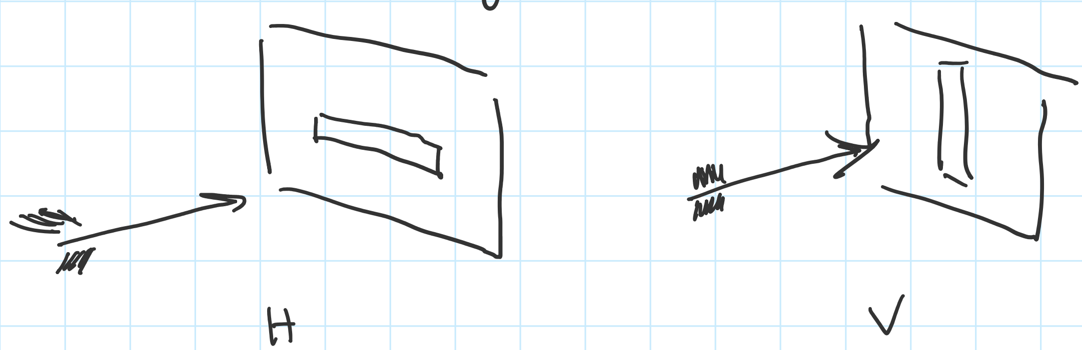
"H" and "T" are <u>states</u> that can be in <u>superposition</u>,

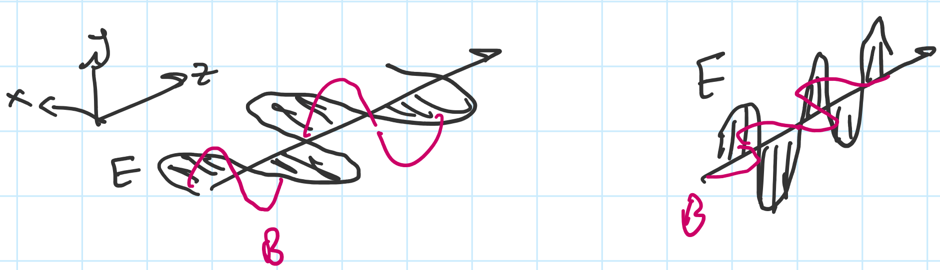
specifically quantum superposition

Therefore, a qubit can be made from any physical system that has two states that can be in quantum superposition

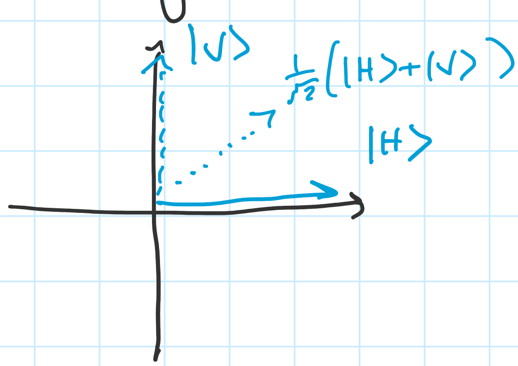
NB: I make this distinction from "two-level system" b/c that conjures up an image of energy levels, like in a hydrogen atom. That is one implementation of qubits, but it is not the only one! Again, most generally it is two <u>states</u>. We will see this in our first example.

- Polarization: a visual qubit (~5 min)



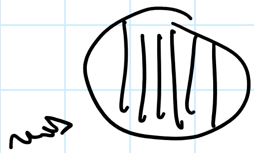


Literally, a flying vector



Measurement

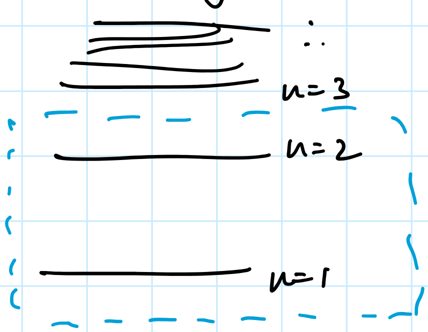
"Analyzer"

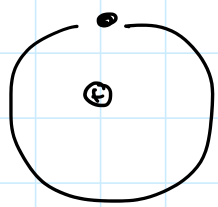


H analyzer $= |H\rangle\langle H| \doteq \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

V analyzer $= |V\rangle\langle V| \doteq \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

- Two-level systems (~5 min)

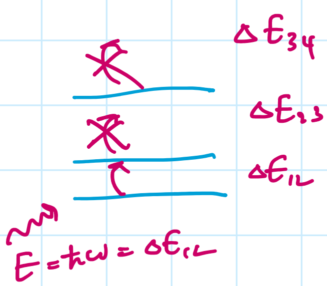




$$E_n = -\frac{13.6\,eV}{n^2}$$

"Anharmonic" — unequal energy spacings

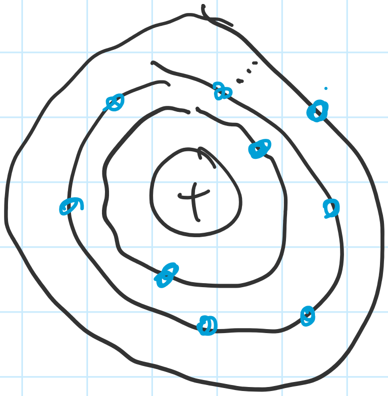
Q: Why? (helps isolate two levels of interest)



Other transitions off-resonant! Blocked!

a) Neutral atoms (e.g. QuEra) (~15 min)

Rb-87

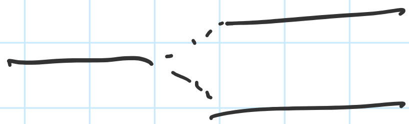


Bohr model

↓

Encode in pair of energy levels

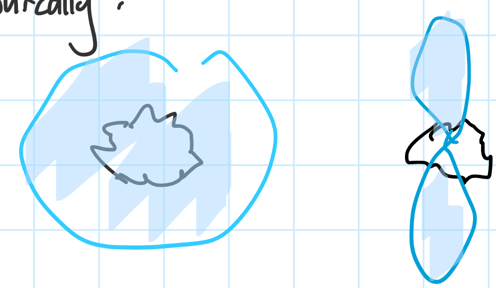
$5S_{1/2}$ $(n=5, \ell=0, j=1/2)$



"hyperfine splitting"

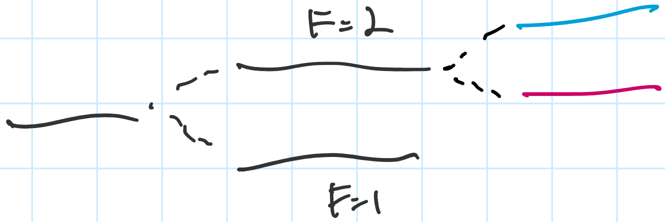
Physics: interaction b/w nucleus and electron clouds

Graphically:



Seems like overlap would be different, intuitively interactions would have different strength

Add magnetic field: Spin as "tiny bar magnet"
^ static



$|0\rangle = |F=1, m_F=0\rangle$

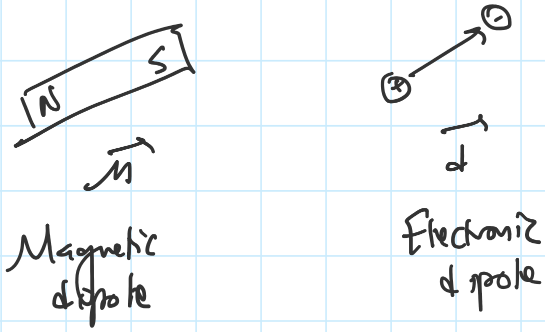
$|0\rangle = |F=1, m_F=0\rangle$

$|1\rangle = |F=2, m_F=0\rangle$

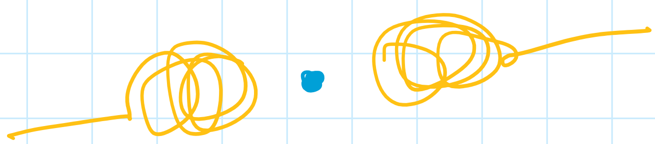
How to drive transitions?

1) Need to match energy

2) Need to inject energy in a way qubit is sensitive to



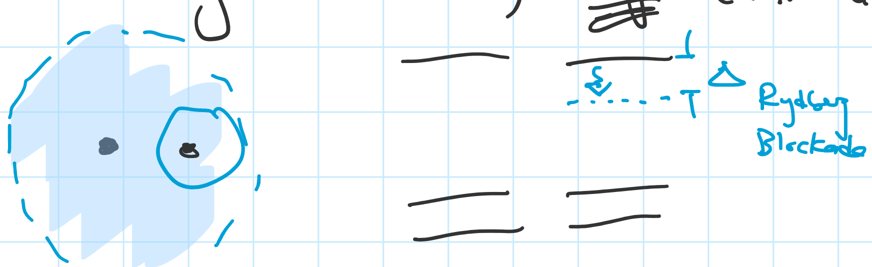
Either way, same method: RF field



$$f_{RF} = \frac{\Delta E_{Rb}}{h} \quad (\sim \text{GHz})$$

Rydberg atoms

→ very excited state, huge electron cloud!



→ super strong atom-atom interactions can enable entanglement can enable entanglement

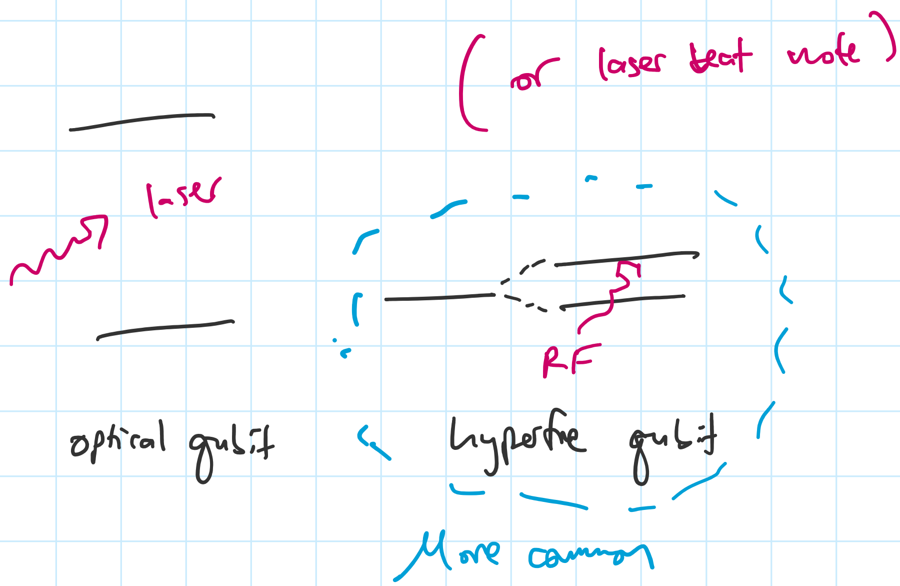
(Specifics: interaction slightly shifts non-Rydberg atom energy levels, detuning from efficient driving)

b) Trapped ions (e.g. IonQ) (~5 cm)

See slide, very similar to neutral atoms

e.g. Yb^+, Ca^+ (prepped by laser ionization)

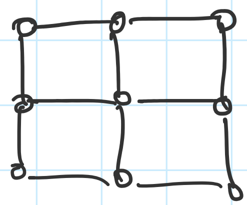
oscillating RF field traps the ions (compared to optical tweezers for neutral atoms)



c) Spin defects in solids (e.g. Light Synq) (~10 mm)

c) Spin defects in solids (e.g. Light Synq) (~10 min)

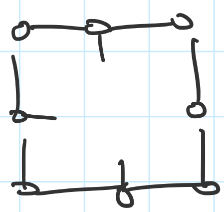
- See slide, control similar to atoms
- NV demo?



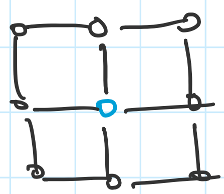
← perfect lattice

Thermodynamically unstable!

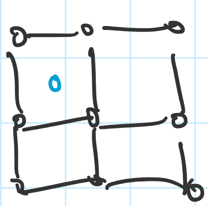
Defect = Actual lattice − Perfect lattice



Vacancy



Impurity



Interstitial

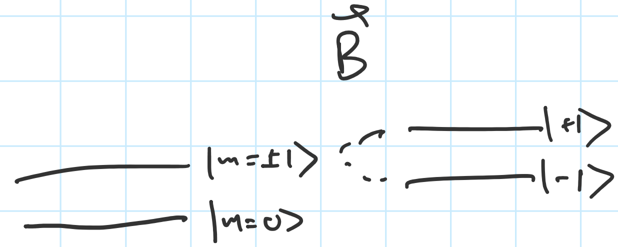
And more!

Point defects — "like an artificial atom trapped in a crystal!"

e.g. NV = "nitrogen vacancy center"

e.g. NV = "nitrogen vacancy center"

V_{Si} = "silicon vacancy"



Split w/ magnetic field, drive w/ RF ... etc

Cool thing! Memory qubits (couple to nuclear spin)

↳ very long-lived / shielded = store quantum info!

(IF they are close by)

Enough with the atom-sized qubits

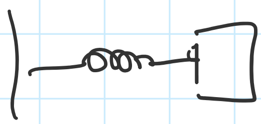
↳ macroscopic quantum behavior?

Enter ... superconducting qubits

Intro ... superconducting qubits

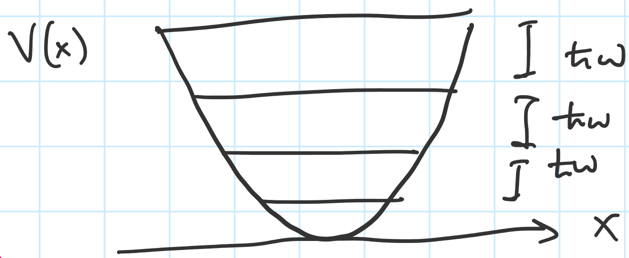
d) Superconducting circuits

Harmonic oscillators (springs!) BONUS



Energy = Kinetic + Potential

$$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$



Equal spaced quantized energy levels!

(Note: will not prove)

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

Light: also a harmonic oscillator!

(Hence, $E = hf$ per photon)

Energy = Magnetic + Electric

$$= \frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2$$

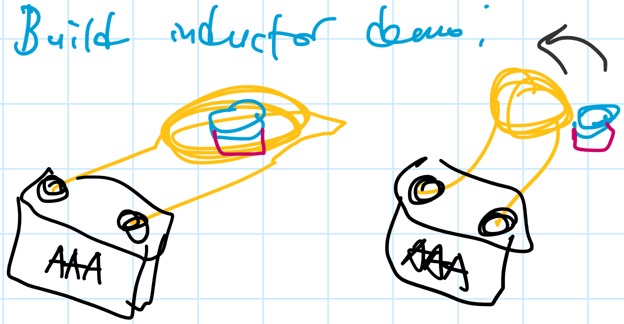
LC circuit: also a harmonic oscillator!!!

Energy = Magnetic + Electric

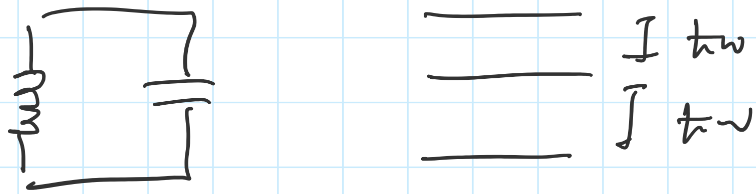
$$= \text{Inductive} + \text{Capacitive}$$

$$= \frac{1}{2} L I^2 + \frac{1}{2} C V^2$$

Build inductor demo:



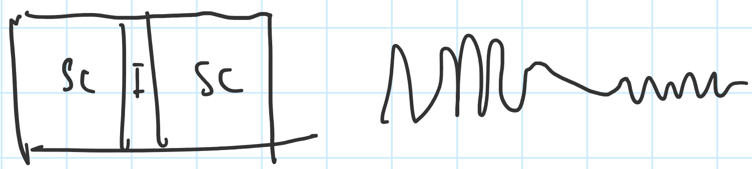
Electromagnets!



Wait... but equal spacing (harmonic) is bad!

Need to introduce anharmonicity

↳ Josephson Junction — a nonlinear inductor

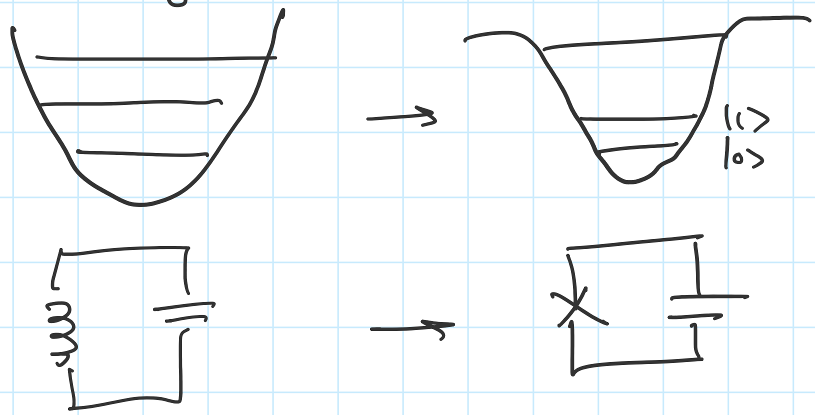


$V \propto \frac{dI}{dt}$, proportionality called "inductance"

NB: this inductance does not come from stored magnetic energy; rather, from the inertia of the Cooper pairs opposing a change in motion

→ Anharmonicity from nonlinearity

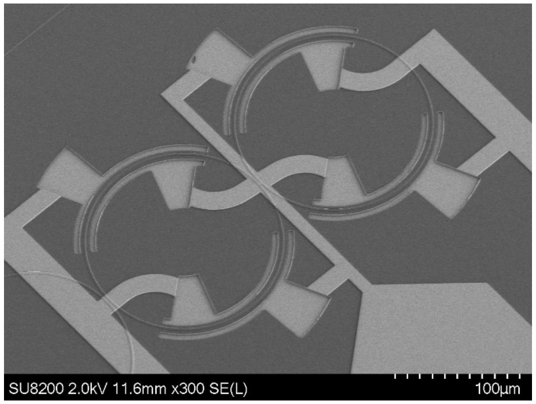
→ Anharmonicity from nonlinearity



From here, the usual techniques follow — apply RF pulses, etc.

Emphasize: trying to engineer world that is very small
But many of the same principles apply

Physical implementations of qubits



Matthew Yeh
Adapted from slides given by
Ben Pingault, ANL

2025, MIT Quantum Winter School

Quantum computation: the quantum bit

- Two-state quantum system: possible superposition of 0 and 1

$$|\psi\rangle = c_1|0\rangle + c_1|1\rangle, \qquad c_0, c_1 \in \mathbb{C} \qquad \langle 0|1\rangle = 0$$

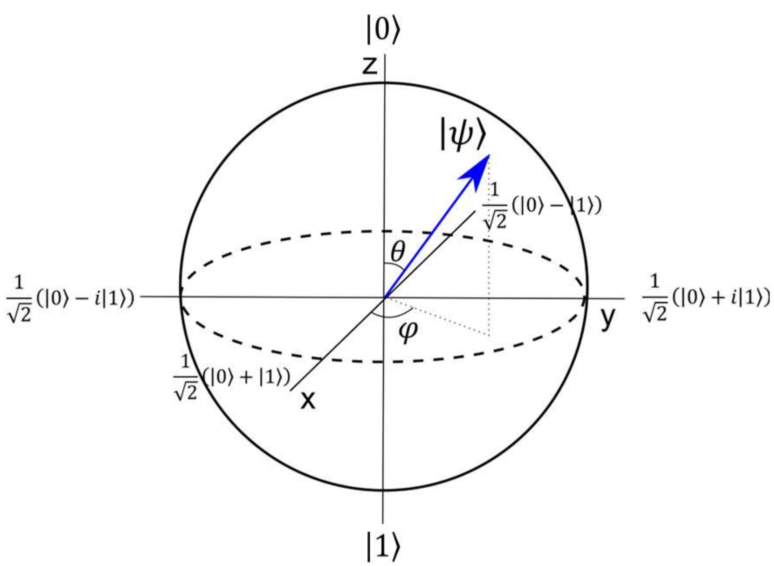
- Information stored in c_0 and c_1:

$$\langle\psi|\psi\rangle = |c_0|^2 + |c_1|^2 = 1$$

$$|\psi\rangle = \binom{c_0}{c_1}$$

- Representation as a Bloch vector:

$$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\varphi}\sin(\theta/2)\,|1\rangle$$



DiVincenzo criteria

Set of 5 requirements for a physical system to be used as a suitable qubit:

- Scalability (well-characterised qubit)
- Simple initialization
- Coherence time much longer than gate operation time
- Single- and two-qubit gates
- Measurement of state of each qubit

- No current system fulfills all requirements
- Main issue: scalability
- Fidelity of operations (initialization, gates, measurement)

DiVincenzo criteria

Set of 5 requirements for a physical system to be used as a suitable qubit:

– **Scalability (well-characterised qubit)**

– Simple initialization

– Coherence time much longer than gate operation time

– Single- and two-qubit gates

– Measurement of state of each qubit

Today's topic!

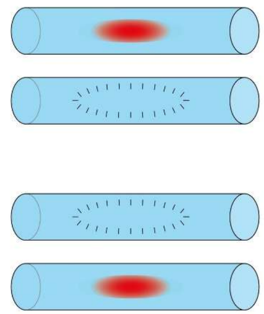
- No current system fulfills all requirements
- Main issue: scalability
- Fidelity of operations (initialization, gates, measurement)

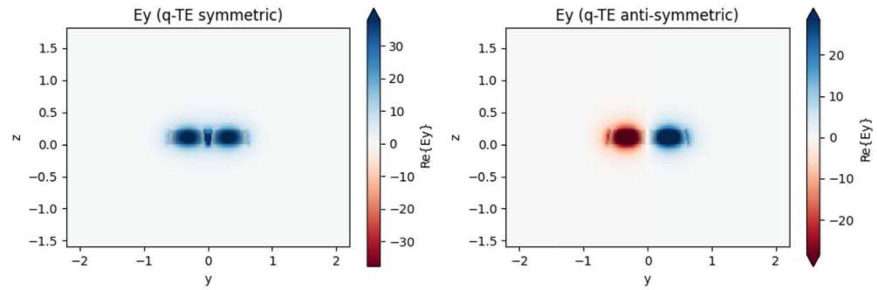
Photons

dual-rail encoding

Qubit:

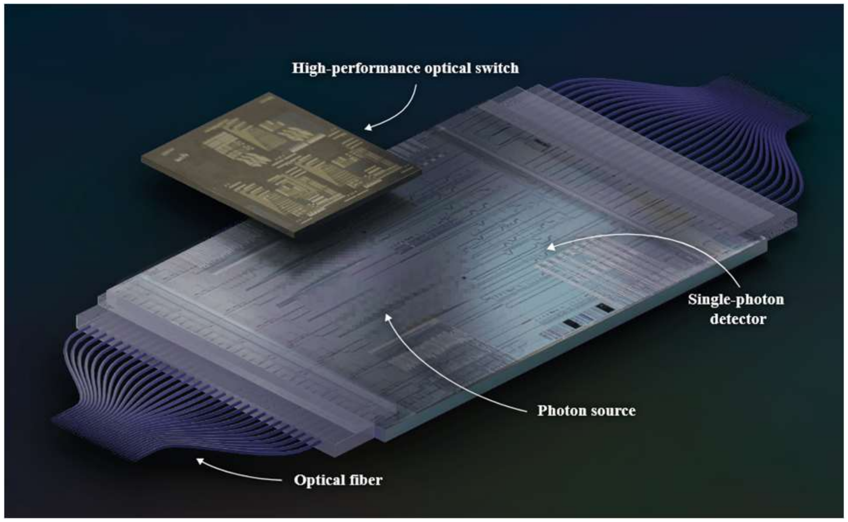
- Polarisation
- spatial mode
- temporal mode (time bin: early, late)
- spectral mode (frequency bin: red, blue)





PsiQuantum

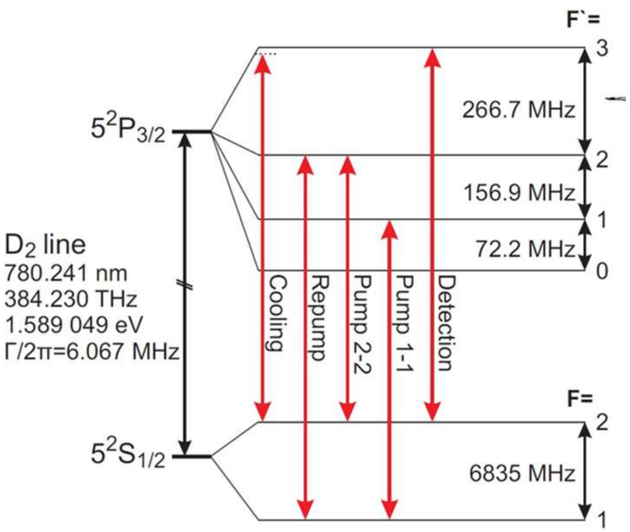
XANADU

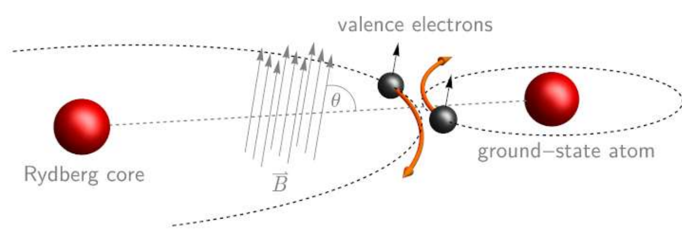


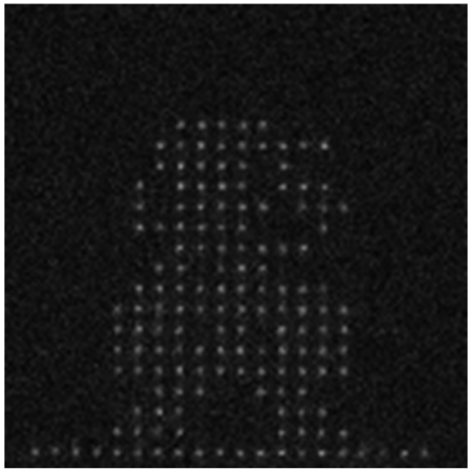
Neutral atoms

Qubit:

- Hyperfine states of neutral atoms
- Rb, Cs



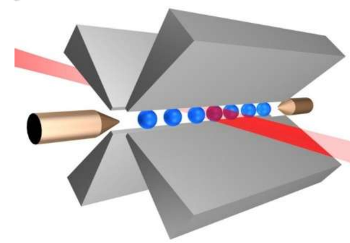


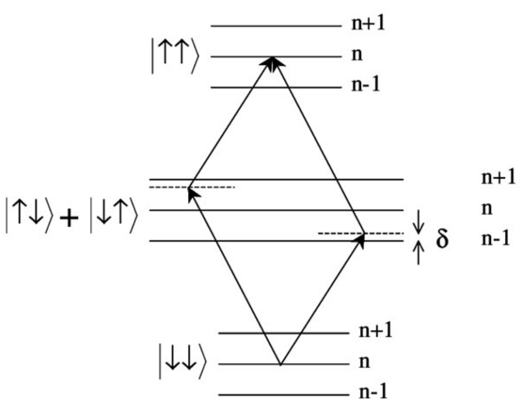


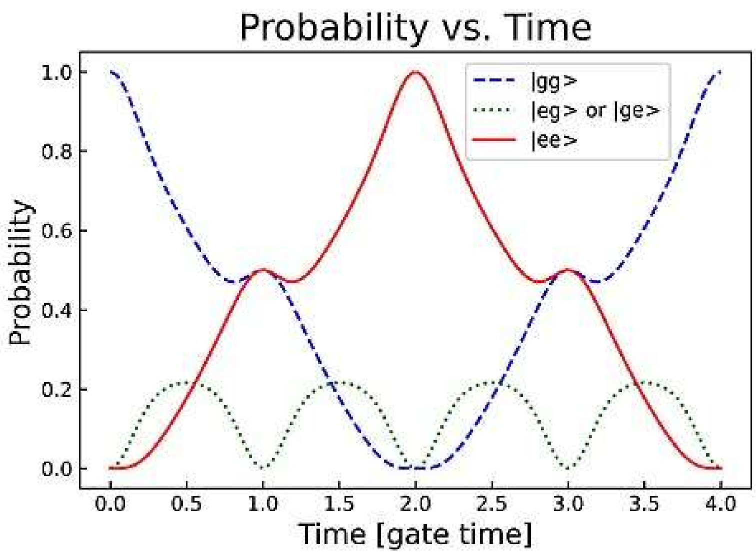
Trapped ions

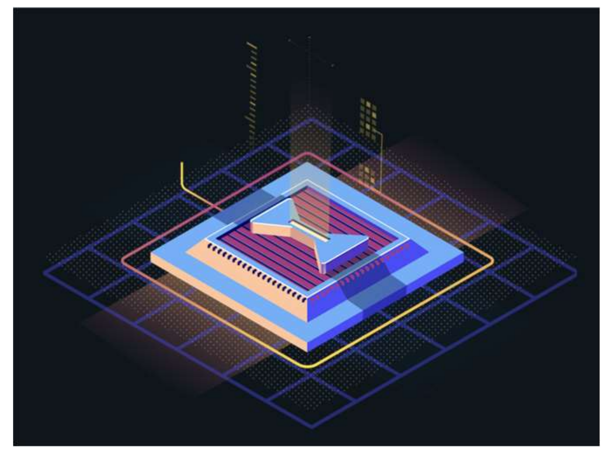
Qubit

- Hyperfine states of charged atoms
- $^{40}Ca^{+}$, $^{171}Yb^{+}$





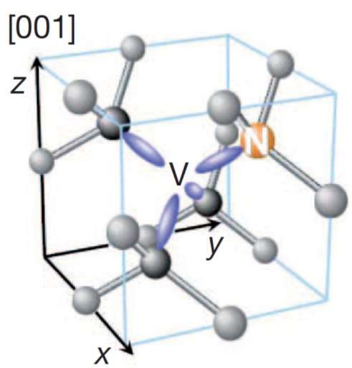


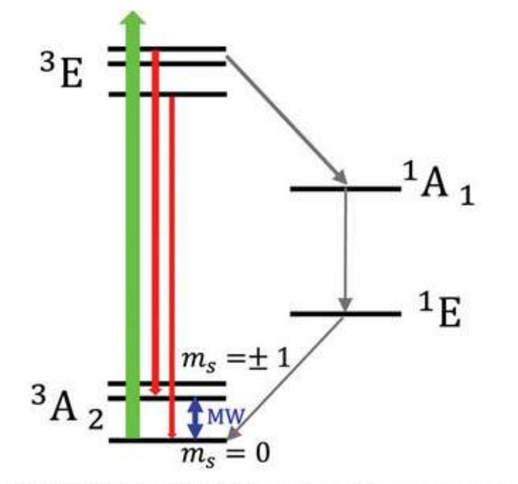


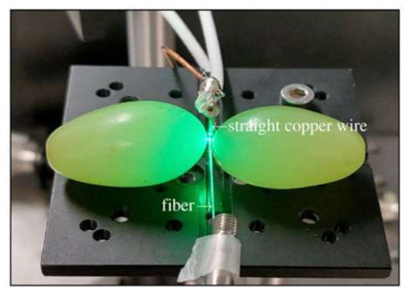
Defects in solids

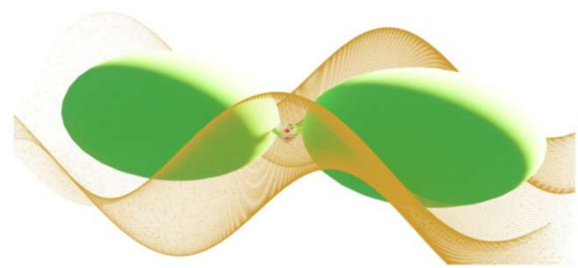
Qubit

- Magnetic sublevels of spin defect
- Diamond: NV, SiV, SnV, PbV…
- SiC: V_{si}, VV
- Si: T center









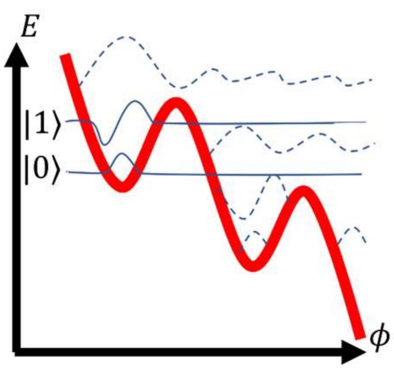
[34] Steger M, *et al., Science* **336**, 1280 (2012)
[35] Abobeih M, et al., Nature Commun. 9, 2552 (2018)

Superconducting qubits

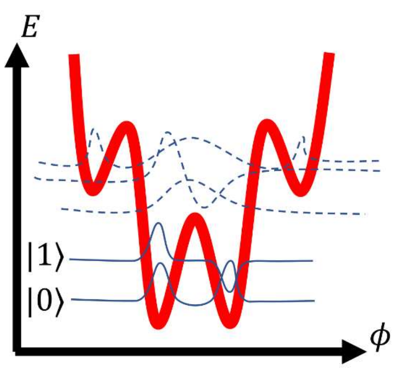
Qubit

- Phase qubit: ’phase particle’ energy levels
- Flux qubit: cw and ccw supercurrent
- Charge qubit: Cooper pair charge

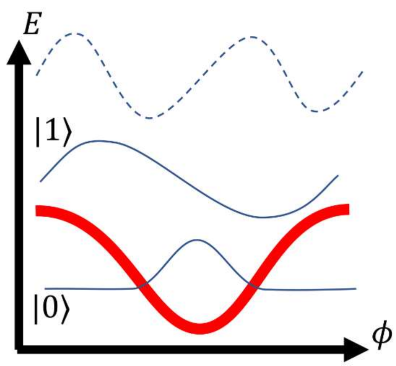
phase



flux



charge



G Wendin, Rep. Prog. Phys.80 106001 (2017)
Houck et al, Quantum Information Processing 8, 105–115 (2009)

Problem-solving session mini-lecture.

(1) inner + outer products.

We've learned what a $|\cdot\rangle$ "ket" vector is.

Dual to a "ket" is "bra", which is just its conjugate-transpose: $\langle\cdot| = (|\cdot\rangle)^\dagger$

don't worry about this now.

That is, since $|\cdot\rangle$ is a column vector, $\langle\cdot|$ is a row-vector:

$$\langle 0| = (1 \quad 0)$$

$$\langle 1| = (0 \quad 1)$$

We can do some operations on these vectors:

Inner-product

$$|\psi_A\rangle = \alpha_A|0\rangle + \beta_A|1\rangle$$

$$|\psi_B\rangle = \alpha_B|0\rangle + \beta_B|1\rangle$$

$$\langle\psi_A|\psi_B\rangle = \begin{pmatrix}\alpha_A^* & \beta_A^*\end{pmatrix}\begin{pmatrix}\alpha_B \\ \beta_B\end{pmatrix}$$

$$= \alpha_A^*\alpha_B + \beta_A^*\beta_B$$ → scalar $a \in \mathbb{R}$

some properties:

if $|\psi_A\rangle \equiv |\psi_B\rangle$ then $\langle\psi_A|\psi_B\rangle = 1$

if $|\psi_A\rangle \perp |\psi_B\rangle$ then $\langle\psi_A|\psi_B\rangle = 0$.

Outer-product:

$$|\psi_A\rangle\langle\psi_B| = \begin{pmatrix}\alpha_A \\ \alpha_B\end{pmatrix}\begin{pmatrix}\alpha_B^* & \beta_B^*\end{pmatrix}$$

fix notation, use 1,2 not A, B.

$$= \begin{pmatrix}\alpha_A\alpha_B^* & \alpha_A\beta_B^* \\ \alpha_B\alpha_B^* & \alpha_B\beta_B^*\end{pmatrix}$$

→ creates matrices.

(ii) Hilbert space:

First need to define a vector space:

A set V is a vector space if:

1. $\exists$ a function f that maps each pair of elements $u, v \in V$ to an element $u + v \in V$.
2. $\exists$ a function f that maps each $v \in V$ to an element $\lambda v \in V$, for each scalar λ.

— AND —

some special properties hold.

Sorry this is dry, you've gotta eat your vegetables!

A <u>Hilbert space</u> is a finite-dimensional vector space equipped w/ an inner product (for our purposes).

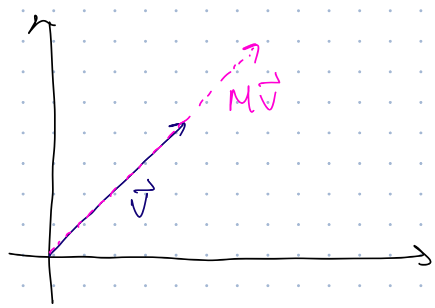
(iii) Eigenvalues + eigenvectors:

eigenvector of a matrix M:

a vector $\vec{v}$ such that

$$M\vec{v} = \lambda \vec{v}, \text{ where } \lambda \text{ is a scalar.}$$

Why do we care? For an observable M, the eigenvalues are the possible measurement outcomes.



(iv) Expectation value:

The expected value of a measurement outcome.

Given an operator A and a state $|\psi\rangle$,

the expected value $\langle A \rangle = \langle \psi | A | \psi \rangle$.

Written using the spectrum of A:

$$A = \sum_j \lambda_j |v_j\rangle\langle v_j|$$

$$\langle A \rangle = \sum_j \lambda_j |\langle \psi | v_j \rangle|^2$$

Explain: expected value is the outcome you would get from measuring ∞ times!